Exam Game Theory (191521800)
University of Twente
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This exam has 5 exercises.
Motivate all your answers! You may not use any electronic device.
This exam comes with a cheat sheet that contains most of the basic definitions. (See the last three pages.) Other necessary definitions are given in the questions. You are also allowed to bring your own cheat sheet (1 A4).

Noncooperative Game Theory

1. (12 points) Consider the bimatrix game given by $A = \begin{pmatrix} -1 & 14 \\ 7 & 12 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 9 \\ 16 & 14 \end{pmatrix}$.

   a) [3 points] Compute all Nash equilibria of this game.

   b) [3 points] Write down all conditions that define the correlated equilibria of this game, and exhibit a correlated equilibrium that is not a Nash equilibrium.

   c) [3 points] Consider a game G with three players 1,2, and 3 with corresponding strategy sets S_1, S_2 and S_3, *resp.* Let $\tilde{G}$ denote the game where players 2 and 3 are combined, so $\tilde{G}$ has only two players with strategy sets S_1 and $S_2 \times S_3$, *resp.* What can you say about the relation between Nash equilibria in G and $\tilde{G}$? In particular, assume $(s_1, s_2, s_3) \in S_1 \times S_2 \times S_3$ is a Nash eqilibrium in G. Can you conclude that $(s_1, (s_2, s_3))$ is a Nash equilibrium in $\tilde{G}$? And what about the converse?

   d) [3 points] Same question as in c), but then for correlated equilibria instead of Nash equilibria.

Cooperative Game Theory

2. (9 points) Consider the following three-person cooperative game (N, v). (This is actually a cost game. For convenience all costs are written as negative profits.)

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	-3	-4	-5	-5	-6	-6	-7

   (a) Show that this game is convex.

   (b) Determine the core $C(v)$.

Please turn over.

A specific solution for a cooperative game (N, w) is the allocation $y(w)$, which allocates the total profit $w(N)$ proportional to the square of the individual values,

$$y_i(w) = \frac{w(\{i\})^2}{\sum_{j \in N} w(\{j\})^2} w(N), \quad i \in N.$$

(d) Prove that $y(w)$ is efficient and symmetric for any game (N, w).

(e) Calculate $y(v)$. Does it belong to the core?

3. (3 points) Consider the T-unanimity game u_T. Prove that the Shapley value of this game equals $\phi(u_T)$ with $\phi_i(u_T) = 1/|T|$ if $i \in T$ and $\phi_i(u_T) = 0$ else.

Stochastic Game Theory

4. (8 points) Consider the following stochastic game situation with discount factor 0.8 and infinite horizon:

2 $(\frac{1}{2}, \frac{1}{2})$	4 $(1, 0)$
3 $(1, 0)$	3 $(\frac{1}{2}, \frac{1}{2})$

$s = 1$

2 $(0, 1)$

$s = 2$

(a) Describe an MR-decision rule and a HD-decision rule for this game. Briefly explain the difference between these types of rules.

(b) Determine the value v_β of this game and optimal strategies of the players.

(c) Instead of an infinite horizon, let the horizon length be $T = 1$. The salvage value is $e(s) = v_\beta(s)$. Prove that the value of this finite horizon game equals the value of the infinite horizon game, that is, $v_{\beta,1} = v_\beta$.

5. (4 points) Consider a two-players dice game in which players accumulate points by turns with the following rules. Each of two players, by turns, rolls a dice several times accumulating the successive scores until he decides to stop, or he rolls an ace (1). When stopping, the accumulated turn score is added to the player account and the dice is given to his opponent. If he rolls an ace, the dice is given to the opponent without adding any point. The player who first reaches 100 points is the winner of the game.

Model this game as a transient stochastic game.

Total: 36 + 4 = 40 points