Exam Game Theory (191521800)

University of Twente

November 09, 2023, 8:45-11:45h

This exam has 9 exercises.
Motivate all your answers! **You may not use any electronic device.**
You are allowed to bring your own cheat sheet (1 A4, double-sided).

Noncooperative Game Theory

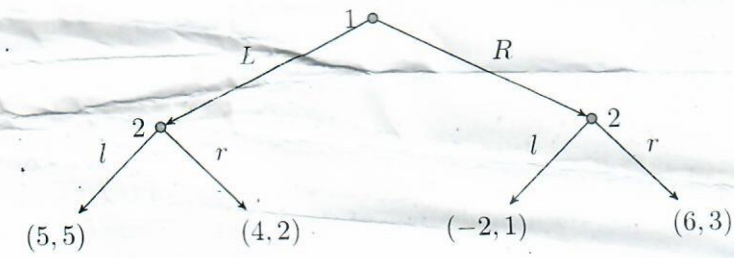
1. (2+2 points) Consider the bimatrix game given by

$$(A,B) = \begin{pmatrix} -2,1 & 14,9 \\ 6,16 & 12,14 \end{pmatrix}.$$

   (a) Compute all Nash equilibria of this game.

   (b) Write down all conditions that define the correlated equilibria of this gamen and give a correlated equilibrium that is not a Nash equilibrium.

2. (1+2 points) Consider the following extensive form game (with perfect information and perfect recall).



   (a) Give the strategic form representation of this extensive form game

   (b) Compute all subgame perfect equilibria.

3. (3+2 points) We denote by

$$C_x := \{P \mid P \text{ is a correlated equilibrium in which the expected payoff of player 1 is } x\}$$

and

$$N_x := \{\sigma \mid \sigma \text{ is a Nash equilibrium in which the expected payoff of player 1 is } x\}.$$

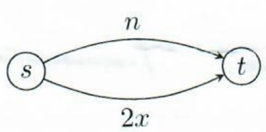
Prove or disprove:

   (a) C_x is a convex set for all $x \in \mathbb{R}$.

   (b) N_x is a convex set for all $x \in \mathbb{R}$.

(Hint: We know that the set of correlated equilibria is convex.)

4. (2 points) Consider the simple road network shown below.



There are n cars going from s to t. The upper road takes n minutes (independent of the number of cars that use this road), the lower road takes $2x$ minutes if x cars take it.
What is the price of anarchy for this instance? (You may assume that $4|n$).

Cooperative Game Theory

5. (1 + 3 + 2 points) Consider the following three player cooperative game (N, v).

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	1	2	5	14	15	11	20

   (a) Is the game essential? Is it superadditive? Is it convex?

   (b) Compute the core $C(N, v)$, and expres it as convex hull of its extreme points. What is the domination core $DC(N, v)$ for this game?

   (c) Compute the Shapley value ϕ. Is $\phi \in C$? Is the Weber set $W = C$?

6. (4 points) Consider a simple game (N, v) with $|N| = n$ and $1 \leq m \leq n$ veto players which we assume to be the players $1, \ldots, m$. Let x be the allocation with $x_i = \frac{1}{m}$ for $1 \leq i \leq m$ and $x_i = 0$ otherwise. Show that x is the nucleolus.

Stochastic Game Theory

7. (3 points) Consider the repeated game $G^\infty(\delta)$ with

$$G = \begin{array}{c} \\ T \\ B \end{array} \begin{array}{c} \begin{array}{cc} L & R \end{array} \\ \begin{pmatrix} 1,6 & 3,4 \\ 2,3 & 4,1 \end{pmatrix} \end{array}$$

   For which values of δ is (T, R) a subgame perfect equilibrium of $G^\infty(\delta)$?
   (Hint: (B, L) is the unique Nash equilibrium of the bimatrix game G.)

8. (5 points) Consider the following zero-sum stochastic game with an infinite horizon and the discounted-reward criterion with discount factor $\beta = 9/10$.

State 1:

	20 $(\frac{2}{3}, \frac{1}{3})$	60 $(\frac{1}{3}, \frac{2}{3})$
	30 $(\frac{1}{3}, \frac{2}{3})$	10 $(\frac{2}{3}, \frac{1}{3})$

state 1

State 2:

30 $(0, 1)$

state 2

   (a) Determine the value of this game and optimal strategies for the players.

   (b) Show that for $\mathbf{v} \in \mathbb{R}^N$ and a pair of stationary strategies $(\mathbf{f}, \mathbf{g})$ such that

$$\mathbf{v} \geq (1 - \beta)\mathbf{r}(\mathbf{f}, \mathbf{g}) + \beta P(\mathbf{f}, \mathbf{g})\mathbf{v}$$

   it holds that $\mathbf{v} \geq \mathbf{v}_\beta(\mathbf{f}, \mathbf{g})$.

9. (4 points)

   (a) Mention one difference and one similarity between finite-horizon discounted stochastic games and infinite-horizon zero-sum average reward stochastic games.

   (b) Give a numerical example of a two-state stochastic game with the average reward criterion that is terminating.

Total: 36 points. Grade = (points + 4) / 4