Exam MSc Course Game Theory (191521800)

November 7, 2013

Motivate all your answers.

1. (3 points) Consider a matrix game $A \in \mathbb{R}^{m \times n}$. Let, as usual, $v_1 = \max_p \min_q pAq$ and $v_2 = \min_q \max_p pAq$ be the maximin and minimax values for players 1 and 2 respectively. Show that $v_1 \leq v_2$ (not making use of von Neumann's theorem which says that $v_1 = v_2$).

2. (5 points) Consider the (symmetric) bimatrix game given by

$$(A, B) = \begin{pmatrix} -10, -10 & 0, 5 \\ 5, 0 & -1, -1 \end{pmatrix}$$

   (a) Compute all Nash equilibria of this game.

   (b) Write down all conditions that define the correlated equilibria of this game, and give a correlated equilibrium that is not a Nash equilibrium.

3. (4 points) Consider the following 3-player extensive form game. Give the strategic form[1] for this game, and compute the subgame perfect equilibria.

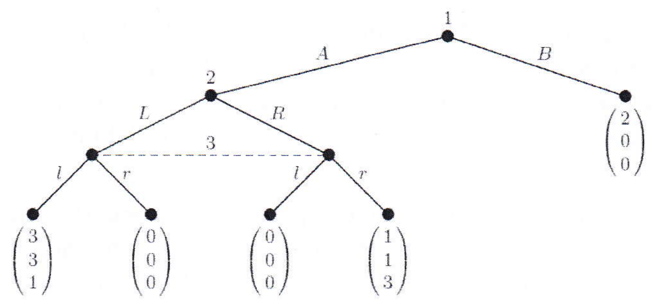

Figure 1: 3-player extensive form game.

4. (8 points) Consider the following three player cooperative game (N, v).

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	1	2	5	14	15	10	20

   (a) Is the game a convex game?

   (b) Compute the core $C(N, v)$, and the domination core $DC(N, v)$. Are they equal?

   (c) What is the maximal value of $v(\{2\})$ such that the core still is nonempty?

   (d) Compute the Shapley value by using the characterization with Harsanyi dividends.

5. (4 points) Let $(\{1, 2, 3\}, v)$ be a three-person game which has a nonempty core. Show that $2v(\{1, 2, 3\}) \geq v(\{1, 2\}) + v(\{1, 3\}) + v(\{2, 3\})$.

[1] This is a 3-player game, so you may give two 2×2 payoff matrices, one for the case that player 1 plays A and one for the case that player 1 plays B.

6. (4 points) Consider the (infinitely) repeated game $G^\infty(\delta)$, with discount factor δ and

$$G = \begin{array}{c} \\ T \\ B \end{array} \begin{array}{c} \quad T \qquad\quad B \\ \begin{pmatrix} 15,13 & 9,14 \\ 20,8 & 10,10 \end{pmatrix}. \end{array}$$

Consider the strategy $Tr(T)$: at $t = 0$ and every time t such that in the past only (T, T) has occurred in the stage game, play T. Otherwise, play B.

For which values of δ is $(Tr(T), Tr(T))$ a subgame perfect Nash equilibrium?

7. (4 points) One special class of stochastic games are additive reward and additive transition (AR-AT) games. For such games, the rewards and transitions may be written as the sum of a term determined by player 1, and a term determined by player 2. That is, $r(s, a^1, a^2) = r(s, a^1) + r(s, a^2)$, and $p(s'|s, a^1, a^2) = p(s'|s, a^1) + p(s'|s, a^2)$. Below we describe a situation with this AR-AT structure.

   Consider two fishing companies, which both have their concessions to catch fish. The availability of fish the next year is determined by the aggregate greed of the companies in the preceding year. They should leave enough fish in the ocean to take care of next generations of fish. Thus, both companies additively contribute to the change in the state, that is, the availability of fish.

   Further, the payoffs are also of the additive type. Namely, the costs of a company depend on a number of company dependent factors, like efficiency, cost of its equipment, and the amount of fish it decides to catch, while its revenues depend on the price of fish, which can be assumed to be a function of the total availability of fish, that is, of the state of the system.

   In this setup, when one thinks of a competition between the firms for obtaining a market share as high as possible, a zero-sum stochastic game might be an appropriate representation. Model this fishery situation as an AR-AT stochastic game, and give a small numerical example.

8. (4 points) Consider a discounted stochastic game. Let $\mathbf{v} \in \mathbb{R}^n$, and let $(\mathbf{f}, \mathbf{g})$ be a pair of stationary strategies such that $\mathbf{v} \geq (1 - \beta)\mathbf{r}(\mathbf{f}, \mathbf{g}) + \beta P(\mathbf{f}, \mathbf{g})\mathbf{v}$. Show that $\mathbf{v} \geq \mathbf{v}_\beta(\mathbf{f}, \mathbf{g})$.

Total: 36 + 4 = 40 points