Exam MSc Course Game Theory (191521800)
February 7, 2014; University of Twente

Motivate all your answers.

1. (6 points) Consider the bimatrix game given by

$$(A, B) = \begin{pmatrix} 4,4 & 1,5 \\ 5,1 & 0,0 \end{pmatrix}$$

   (a) Compute all Nash equilibria of this game.

   (b) Briefly describe in your words, what is the difference between Nash and Correlated equilibria?

   (c) Write down all conditions that define the correlated equilibria of this game, and give an example of a correlated equilibrium of this game that is not a Nash equilibrium.

2. (6 points) Consider the following two player extensive form game.

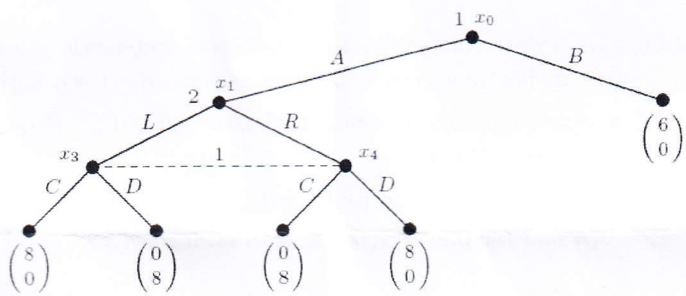

Figure 1: Extensive form game.

   (a) Give the corresponding 3×2 bimatrix game.

   (b) Compute all Nash equilibria of this 3×2 bimatrix game, and also give the corresponding behavioral strategy.

   (c) Compute the subgame perfect equilibria for this game, and briefly discuss the outcome.

3. (8 points) Consider the cooperative game (N, v) with $N = \{1, 2, 3\}$ and $v = 2u_{\{1,2\}} + 6u_{\{1,3\}}$.

   (a) Which players are null players, if any?

   (b) Show that the core of this game is

$$C(v) = \{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 8,\ 0 \leq x_2 \leq 2, 0 \leq x_3 \leq 6\}.$$

   (c) Calculate the nucleolus of this game.

   (d) Does the nucleolus belong to the Weber set?

4. (4 points) Let (N, v) be an essential game. (That is, $v(N) \geq \sum_{i=1}^{n} v(i)$.) Prove that the imputation set is the convex hull of the points $\mathbf{f}^1$, $\mathbf{f}^2$, $\ldots \mathbf{f}^n$, where $\mathbf{f}_k^i = v(k)$ if $k \neq i$, and $\mathbf{f}_k^i = v(N) - \sum_{k \in N \setminus \{i\}} v(k)$ if $k = i$.

5. (6 points) Consider the following discounted stochastic game with infinite horizon and discount factor $\beta = \frac{1}{2}$:

3 $(1, 0)$	0 $(0, 1)$
0 $(0, 1)$	2 $(1, 0)$

state 1

0 $(0, 1)$

state 2

   (a) Write down the set of equations that uniquely determine the value vector of the game.

   (b) Determine the value of this game, and optimal strategies for the players.

6. (6 points)

   (a) Mention two (nontrivial) differences between stochastic games with discounted rewards and stochastic games with average rewards (both with infinite horizon).

   (b) Consider a zero-sum β-discounted stochastic game Γ_β. Prove that the following two statements are equivalent:

      (i) (π_*^1, π_*^2) is an equilibrium point in Γ_β.

      (ii) π_*^1 is optimal for player 1, π_*^2 is optimal for player 2, and $\mathbf{v}_\beta(\pi_*^1, \pi_*^2) = \mathbf{v}_\beta$.

Total: 36 + 4 = 40 points