Mathematical Finance (191515201)
Final Examination
January 26, 2012

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1. (a) (5 points) What is the lower bound for the price of a four-month call option on a non-dividend paying stock when the stock price is \$28, the strike price is \$25 and the risk free interest rate is 8% per year?

   (b) (5 points) You see the following European call prices (same maturity):

strike	50	55	60
price	18	15	11

   Show that there is an arbitrage opportunity, no matter what the risk-free rate is. (Hint: Use a butterfly spread)

2. (a) (5 points) Consider a single-period binomial model with

$$u = 1.05, \quad d = 0.95, \quad \Delta t = \frac{1}{12} \text{ (one month)}, \quad S = \$100.$$

   Suppose the stock pays no dividend. For what values of the continuously compounded risk-free rate r is it optimal to exercise an American put with with strike $K = \$103$ early?

   (b) (5 points) A stock price is currently \$50. Its expected return and volatility are 12% and 30%, respectively. What is the probability that the stock price will be greater than \$80 in 2 years? (Hint $S_T > 80$ when $\ln S_T > \ln 80$) → S follows Geometric BM (you need N(x))

3. (5 points) Show that $\frac{e^{(\sigma^2-2r)(T-t)}}{S}$ could be the price of a traded security if S follows the Geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz.$$

   (Hint: A traded security must follow the Black-Scholes partial differential equation)

4. Consider the data in the table below, where all options are European style and expire in one year ($T = 1$). The underlying asset is a stock that pays no dividends prior to expiration, and whose current price is \$100. The continuously compounded risk-free rate is 5% per year.

	call, $K = 100$	put, $K = 100$	call, $K = 90$	put, $K = 90$
price		13.15	22.98	
delta		-0.37	0.72	
gamma		0.0095	0.0084	
vega		38	33	

   (a) (3 points) Fill in the missing values in the above table. Indicate the formulas you use.

   (b) (2 points) Suppose you currently are long two puts with $K = 100$ and short one put with $K = 90$. Compute the delta, gamma and vega of your portfolio.

(c) (3 points) How can you make your portfolio in part (b) both delta and gamma neutral by taking positions in the underlying stock and the \$100-strike call?

(d) (2 points) If you were to increase your estimate of the stock volatility by 1%, by approximately how much would that change the call and put price for the \$100 strike?

5. (a) (5 points) Calculate the price of an option that caps the 3-month rate starting in 15 months' time at 13% (quoted with quarterly compounding) on a principal amount of \$1000. The forward interest rate for the period in question is 12% per year (quoted with quarterly compounding), the 18-month risk-free interest rate (continuously compounded) is 11.5% per year, and the volatility of the forward rate is 12% per year.

   (b) (5 points) Suppose that $a = 0.1$ and $b = 0.1$ in the Vasicek model. Assume that the initial short rate is 10% and the initial standard deviation of the short rate change in a short time Δt is $0.02\sqrt{\Delta t}$. Calculate the price of a zero-coupon bond that matures in 10 years.