Exam: Optimization Modelling

Code : 158142
Date : January 25, 2007
Preliminaries: 1. You may answer in Dutch. 2. Calculators are not allowed. 3. Every answer needs to be motivated!

1 Either-or Constraints

Consider the following model:

$$\begin{aligned}
&\textit{minimize} && \sum_{j \in J} c_j x_j \\
&\textit{subject to}: && \\
&\sum_{j \in J} a_{1j} x_j && \leq b_1 \qquad (1) \\
&\sum_{j \in J} a_{2j} x_j && \leq b_2 \qquad (2) \\
&x_j && \geq 0 \quad \forall j \in J
\end{aligned}$$

where: at least one of the conditions (1) or (2) must hold.

Due to the *either-or*-condition the model is not linear. Formulate a linear programming model that is equivalent to the above. Provide a short argument why your linear model is equivalent to the above.

2 A Fractional Objective

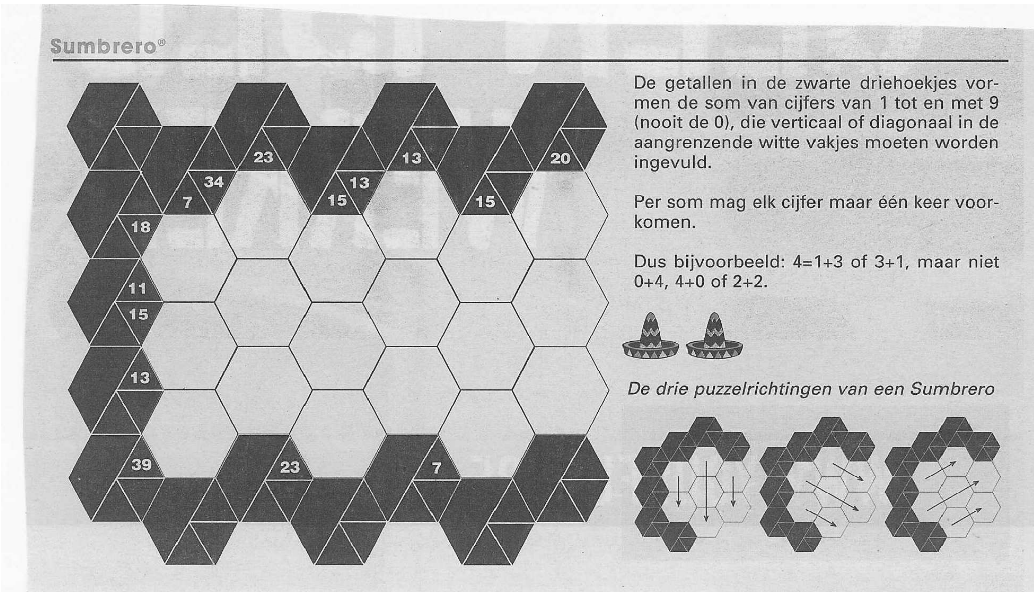
Consider the following model:

$$\begin{aligned}
&\textit{minimize} && \frac{\sum_{j \in J} c_j x_j}{\sum_{j \in J} d_j x_j} \\
&\textit{subject to}: && \\
&\sum_{j \in J} a_{ij} x_j && \leq b_i \quad \forall i \in I \\
&x_j && \geq 0 \quad \forall j \in J
\end{aligned}$$

Due to the *fractional objective* the model is not linear. Formulate a linear programming model that is equivalent to the above, by the introduction of new variables and substitution. Under which conditions is your linear model equivalent to the above?

3 Sumbrero

In the first lecture we constructed a linear programming model for the Sudoku puzzle. An other interesting puzzle, also frequently found in the Metro newspaper, is the Sumbrero. Details on the Sumbrero puzzle is given in the figure below:



Build an linear programming model for the Sumbrero problem.

4 Multi-item lot sizing

Consider the *Multi-item lot sizing* problem which we discussed in one of the lectures. You can find the problem description and the basic linear programming formulation in Appendix A.

1. Argue that it is not necessary to require the variables y_{pt}, z_{pqt} and s_{pt} to be integer, and still have a valid model formulation. Why is this observation useful to us?

2. Which constraints can be dropped if we assume all coefficients of the objective to be positive? Give a short argument why they can be dropped and why this can be useful to us.

3. **Multiple machines**
   In the basic problem there was only one machine to be scheduled. Lets

suppose that there are multiple machines available for production, and each machine can producing all types of product. However, not each machine is equally suitable for each product, this is reflected in the production rate, the minimum run and down times, and all costs (except storage costs). Write down an optimization model that can handle this multi-machine case.
(Tip: For your and my convenience, base your model/notation on the one in the appendix, and make use of the observations in question 4.1 and 4.2)

4. **Changeover times**
   Upto now we have assumed that the changeover times where negligible. Lets suppose they are not. For each machine there is a changeover time of one day.
   You are asked to incorporate this in the multi-machine case of question 4.3.
   (Tip: Perhaps it is useful to read question 4.5 before answering this question.)

5. **Changeover crew**
   The actual change the production on a machine is performed by a crew of technicians. There is only one crew available, this implies that there can be no two machines in *changeover* at the same time.
   Incorporate this in the model you made for question 4.4.

5 Operation Room Scheduling

In the guest lecture Jeroen van Oostrum talked about *A master surgical scheduling approach for cyclic scheduling in operating room departments.* For the selection of an operating room day schedules (ORDS) he considers not all possible ORDS's but uses column generation.

1. Describe the principle of the column generation technique. How does it work and what are its benefits?

2. At the end of the column generation algorithm Jeroen employs a rounding heuristic, why is this necessary?

Determining your mark

In total you can earn upto 100 points, your mark is the scored point divided by 10. The points are divide over the questions in the following way:

Question	1	2	3	4.1	4.2	4.3	4.4	4.5	5.1	5.2
Points	5	10	30	5	5	10	10	10	10	5

Appendix A: Multi-item lot sizing

Problem description

A set of products has to be produced on a single production line during a planning horizon. This horizon is divided into days.

Production takes place in the form of batches covering one or more days. The length of each batch is not known a priori, but should be determined by an optimization model.

For each product there is a minimum batch size in terms of days. In addition, there must be a minimum number of days between successive batches of the same product. There is a known demand for each product, and production during any particular day can also be used to satisfy the demand for that day. The production rate (quantity per day) for each product is known, and does not change over time.

There is a changeover cost when one batch is ended and a new one is started (due to equipment changes). The actual changing of the equipment needs very little time, and does not affect the level of production of either the ending batch or the starting batch.

Total cost over the planning horizon is determined by the cost to produce the products, to store the products, to start a batch, and to clean production equipment between particular changeovers (the changeover costs).

Basic linear programming formulation

<u>Sets:</u>

p, q	products
t, k	days

<u>Parameters:</u>

α_p	minimum run-time of a batch for product p
β_p	minimum down-time for product p
ρ_p	production rate, products p per day
d_{pt}	demand on day t for product p
e_{pq}	changeover cost when passing from the production of p to q
f_p	production cost for one day producing product p
g_p	start up cost for product p
h_p	storage cost per day per unit of product p

<u>Variables:</u>

x_{pt}	binary, product p is produced on day t
y_{pt}	binary, a batch of product p starts on day t
z_{pqt}	binary, product p is produced on day $t-1$ and q on day t
s_{pt}	integer, stock level of product p at the end of day t

Constraints:

$$
\begin{aligned}
x_{pt} - x_{p,t-1} &\leq y_{pt} && \forall p, t \geq 2 && (1)\\
x_{pt} &= y_{pt} && \forall p, t = 1 && (2)\\
y_{pt} &\leq x_{pt} && \forall p, t && (3)\\
y_{tp} &\leq 1 - x_{p,t-1} && \forall p, t \geq 2 && (4)\\
x_{p,t-1} + x_{q,t} - 1 &\leq z_{pqt} && \forall p, q, t \geq 2,\ with\ p \neq q && (5)\\
z_{pqt} &\leq x_{p,t-1} && \forall p, q, t \geq 2,\ with\ p \neq q && (6)\\
z_{pqt} &\leq x_{q,t} && \forall p, q, t,\ with\ p \neq q && (7)\\
y_{pt} &\leq x_{p,k} && \forall p, t, k,\ with\ t \leq k \leq t + \alpha_p - 1 && (8)\\
y_{pk} &\leq 1 - x_{p,t} && \forall p, t, k,\ with\ t + 1 \leq k \leq t + \beta_p && (9)\\
\sum_p x_{pt} &= 1 && \forall t && (10)\\
s_{pt} &= s_{p,t-1} + \rho x_{pt} - d_{pt} && \forall p, t \geq 2 && (11)\\
x_{pt}, y_{pt} &\in \{0, 1\} && \forall p, t &&\\
z_{pqt} &\in \{0, 1\} && \forall p, q, t, with\ p \neq q &&\\
s_{pt} &\geq 0,\ integer, && \forall p, t &&
\end{aligned}
$$

Objective:

$$
\sum_t \sum_p \left(\sum_{q:q \neq p} e_{pq} z_{pqt} + f_p x_{pt} + g_p y_{pt} + h_p s_{pt} \right)
$$

Constraints (1)-(4) connect the start up variables with the production variables. Constraint (5)-(7) connect the changeover variables with the production variables. Constraints (8)-(9) control the run and down time of a batch. Constraint (10) states that we produce only 1 type of product a day. Finally, constraint (11) takes care of the demand and stock level.