Kenmerk: EWI2014/dmmp/001/BM

Exam Optimization Modeling (191581420)

Tuesday, April 15, 2014, 8:45 – 11:45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of three problems. Please start a new page for every problem.
- Total number of points: 36 + 4 = 40. Distribution of points according to the following table.

1a: 9	2a: 2	3a: 3
1b: 3	2b: 3	3b: 3
	2c: 1	3c: 3
	2d: 2	3d: 3
	2e: 4	

1. Vehicle Routing

You work from home, but have to visit certain clients regularly, namely once every k days. Fortunately, your clients do not care when exactly they are visited – the exact time of day and even the exact days do not matter to them. Their only requirement is there are always exactly $k-1$ consecutive days without a visit.

You can assume that you know the travel times between any pair of clients and to and from home exactly. Of course, you also know k. The time needed for service at the clients is negligible. Given this information, you want to find a schedule for k consecutive days during which every client is visited exactly once. As you care about leisure time and want to keep your evening free as good as possible, your goal is to minimize $\max_{i \in \{1,\ldots,k\}}(t_i)$, where t_i is the travel time on day i.

(a) Build an MILP model for this problem.

(b) Your clients become more and more demanding. They want you to visit them as often as possible. However, you still want to keep your evenings free, which means that you have a time limit for every day.

Build an MILP model that minimizes the number of days needed subject to the constraints that every client is visited exactly once and your total travel time per day is no more than B.

2. Knapsack and Column Generation

We consider the following variant of the knapsack problem: we are given items $1,\dots,n$ and a weight bound B. Item i has a weight of w_i and yields a profit of p_i. There are infinitely many copies of each item. Formulated as an IP, we have

$$\begin{aligned} &\text{maximize } \sum_{i=1}^{n} p_i x_i \\ &\text{subject to } \sum_{i=1}^{n} w_i x_i \le B \text{ and } x_1,\dots,x_n \in \mathbb{N}. \end{aligned}$$

We consider the relaxation

$$\begin{aligned} &\text{maximize } \sum_{i=1}^{n} p_i x_i \\ &\text{subject to } \sum_{i=1}^{n} w_i x_i \le B \text{ and} \\ &\qquad x_1,\dots,x_n \ge 0. \end{aligned} \tag{KS}$$

(a) While the knapsack problem is NP-complete, the relaxation is quite easy to solve. What is the optimal solution to (KS)?

Hint: You can assume that the items are sorted in a way that is convenient for you.

(b) Write down the dual of (KS).

We want to solve (KS) with column generation. Thus, assume that we have a set $S \subseteq \{1,\dots,n\}$ of items whose variables we use.

(c) What is an optimal solution of (KS) restricted to items in the set S? The LP restricted to variables with index in S is given by

$$\begin{aligned} &\text{maximize } \sum_{i\in S} p_i x_i \\ &\text{subject to } \sum_{i\in S} w_i x_i \le B \text{ and } x_i \ge 0 \text{ for } i \in S. \end{aligned}$$

(d) Given your optimal solution from Part (c), what is the corresponding dual solution?

Hint: complementary slackness.

(e) Given your solution of Part (c) and the corresponding dual solution of Part (d), what are the reduced costs of variables $x_1,\dots,x_n$?

When should you add a variable, and which variable should you add? When should you stop and conclude that you have already found an optimal solution for (KS)?

3. Modeling Tricks

(a) Assume that you have fixed numbers $\ell_1 < u_1 < \ell_2 < u_2$. You have a variable x that should only assume a value in the interval $[\ell_1, u_1]$ or in the interval $[\ell_2, u_2]$.

Formulate this behaviour using only linear constraints and appropriate additional (integer) variables.

(b) You are given four binary variables x and z_1, z_2, z_3. Model the behavior

$$x = \begin{cases} 1 & \text{if at least two of } z_1, z_2, z_3 \text{ are } 1, \\ 0 & \text{otherwise} \end{cases}$$

using only linear constraints.

(c) We have variable $x \in \{0, 1, 2\}$ and $y \geq 0$ and fixed values a_0, a_1, a_2. Write the following non-linear constraint using linear constraints (you can also use additional variables):

$$y = \begin{cases} a_0 & \text{if } x = 0, \\ a_1 & \text{if } x = 1, \\ a_2 & \text{if } x = 2. \end{cases}$$

(d) You have two integer variables x and y with range $\{0, \ldots, M\}$ for some large number $M \in \mathbb{N}$. Model the constraint $x \neq y$.