Exam Queueing Theory
Monday, June 18, 2018, 13.30 – 16.30.

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1. Jobs arrive at a machine according to a Poisson process with a rate of 6 jobs per hour. The machine processes the jobs at an exponential rate of 6 jobs per hour. Whenever the total number of jobs in the system becomes larger than 2, a second identical machine is immediately turned on and also starts processing the jobs at an exponential rate of 6 jobs per hour. Whenever the total number of jobs in the system is reduced to 2 again, the machine just finishing a job, is turned off.

   a) Compute the limiting distribution of the number of jobs in the system.

   b) Compute the mean number of jobs waiting in the queue and the mean waiting time of a job in the queue.

   A period during which, uninterruptedly, i machines are simultaneously processing jobs is called an i-period, for $i = 0$, $i = 1$ and $i = 2$ respectively.

   c) Compute the rate (average number of times per hour) at which i-periods occur for $i = 0$, $i = 1$ and $i = 2$.

   d) Calculate the expected duration of an i-period, for $i = 0$, $i = 1$ and $i = 2$.

2. Customers arrive at a server according to a Poisson process with a rate of 1 customer per 6 hours. The server processes customers in order of arrival and the service times consist of two independent exponential phases, both with a mean of 1 hour.

   a) Determine the Laplace-Stieltjes transform of the sojourn time in hours of a customer and use this result to determine the long-run fraction of customers for which the sojourn time is longer than 3 hours.

   b) Determine the mean sojourn time of a job, both by using the result in a) and by using mean value analyis.

   A customer arrives at an arbitrary point in time when the server is busy. Let X denote the total service time of the customer in service at that instant.

   c) Determine the Laplace-Stieltjes transform and the first two moments of X.

3. At a service station the interarrival times of customers are modelled by a mixture of an exponential distribution and an Erlang-2 distribution. More specifically, the interarrival time in minutes is either exponentially distributed with mean 1, with probability $1/4$, or it is Erlang-2 distributed with mean 2, with probability $3/4$.

   The amount of time it takes to serve a customer at the service station is modelled by an exponentially distributed random variable with a mean of 1 minute.

   a) Determine the distribution of the number of customer at the service station at the arrival instant of an arbitrary customer.

b) Show, by using Laplace-Stieltjes Transforms, how the distribution of the waiting time of an arbitrary customer can be derived from the distribution determined in part a) above, and calculate $P(W > 2 \mid W > 0)$.

c) Also give the distribution of the number of customers at the service station at an arbitrary point in time. Clearly explain, how this distribution can be derived from the distribution determined in part a) above.

4. At a server customers arrive according to a Poisson process with a rate of 5 customers per hour. The service times of customers are uniformly distributed with a minimum of 5 minutes and a maximum of 15 minutes. As soon as there are no customers anymore in the system the server leaves to do another activity. After an exponentially distributed time with a mean of 12 minutes he returns. If at that time there are customers present, the server starts serving the customers. However, if at that time the system is still empty the server leaves a second time to do another activity. Also this time he returns after an exponentially distributed time with a mean of 12 minutes. If at that time there are customers present, the server starts serving the customers. If not, the server waits, while doing nothing, until the next customer arrives.

   a) Determine the fraction of time the server serves customers, the fraction of time the server does other activities and the fraction of time the server is doing nothing, respectively.

   b) Determine the mean waiting time of a customer.

   c) Determine the mean number of customers present in the system at an instant that the server starts serving customers and the mean time the server is uninterruptedly serving customers.

5. Cars arrive according to a Poisson process with a rate of 80 cars per hour at a big parking lot. Assume that each car, independently of all the other cars, has with probability 1/2 a 'long' parking time of exactly one hour and with probability 1/2 a 'short' parking time of exactly half an hour. We model the parking lot by an infinite server model.

   a) Give the limiting distribution of the number of cars at the parking lot.

   b) Give the joint limiting distribution of the number of cars at the parking lot with a long parking time and the number of cars at the parking lot with a short parking time.

   c) Give an expression for the transient distribution of the number of cars at the parking lot at time t if we assume that at time 0 the parking lot is empty.

Credits:

1a	b	c	d	2a	b	c	3a	b	c	4a	b	c	5a	b	c	tot
3	3	2	2	3	2	3	3	3	2	3	3	2	2	2	2	
	10				8			8			8			6		40