# UNIVERSITY OF TWENTE.

*Invullen in blokletters/To be completed by student*

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1) a) $\rho$ is fraction of time server is working
amount of work arriving to system per time unit $= \lambda \mathbb{E}B$

b) see reader p 62:

PASTA : $p_n = a_n$

$d_n = a_n$ as number of transitions $n \to n+1$ per time unit must equal number of transitions $n+1 \to n$ per time unit

Hence $d_n = p_n$

c) see reader p 64 for derivation of pgf $P_{Ld}(z)$
and use b) to conclude that $P_L(z) = P_{Ld}(z)$

d) is ex 45 $\quad \tilde{B}(s) = \frac{4}{4+s} \cdot \frac{3}{3+s}$

e) note : service process of d, $\lambda = 1$, $\rho = \lambda \mathbb{E}B = \frac{7}{12}$

insert $\tilde{B}(1-z)$ from d) in c) :

$$P_L(z) = \frac{5}{(z-6)(z-2)}$$

$$\mathbb{E}L = \frac{d}{dz} P_L(z) \Big|_{z=1} = \frac{-5(2z-8)}{(z-6)^2(z-2)^2}\Big|_{z=1} = \frac{-5 \cdot (-6)}{25} = \frac{6}{5}.$$

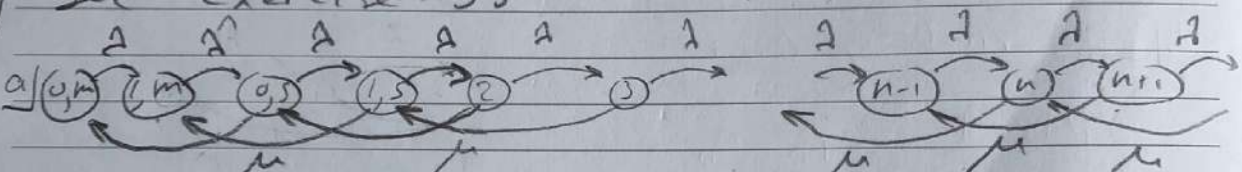
2] see Section 8.3

$$\left.\begin{aligned} \mathbb{E}S &= \mathbb{E}L^a \frac{1}{\mu} + \frac{1}{\mu} \\ \mathbb{E}L^a &= \sum_{n=0}^{\infty} n q_n = \frac{\sigma}{1-\sigma} \end{aligned}\right\} \mathbb{E}S = \frac{1}{(1-\sigma)\mu}$$

(P.116)

$$\Rightarrow \mathbb{E}L = \frac{\rho}{1-\sigma}$$

3] see exercise 55

a] 


$n$ = # waiting for ferry, $(n, m)$ $n$ waiting, ferry moored
$(n, s)$ $n$ sailing
$n = 0, 1$

$\lambda = 15$ / hour
$\mu = 20$ / hour

b] re-label states $q_k$, $k = 0, 1, 2, ..$
balance equations

$$\frac{1}{2}\left\{\begin{aligned} & q_0 \lambda = q_2 \mu \\ & q_1 \lambda = q_0 \lambda + q_3 \mu \\ & q_n(\lambda + \mu) = q_{n-1}\lambda + q_{n+2}\mu \qquad n \geq 2 \\ & \text{trial solution} \quad q_n = x^n \qquad n \geq 2 \\ & \quad \Rightarrow (x-1)\left(x-\tfrac{1}{2}\right)\left(x+\tfrac{3}{2}\right) = 0 \quad \Rightarrow \quad x = \tfrac{1}{2} \end{aligned}\right.$$

$q_n = c x^n \qquad n \geq 2$

boundary eq: $q_0 = c \frac{\mu}{\lambda} x^2$

$q_1 = c \frac{\mu}{\lambda} x^3 + c \frac{\mu}{\lambda} x^2$

normalisation $\sum_{n=0}^{\infty} q_n = 1 \implies c = \frac{3}{4}$

so $p_n$ = prob n car waiting :

$p_n = q_{n+2} = \frac{3}{4}\left(\frac{1}{2}\right)^{n+2} \quad n \geq 2$

$p_1 = q_1 + q_3 = \frac{3}{8} + \frac{3}{4}\left(\frac{1}{2}\right)^3$

$p_2 = q_0 + q_2 = \frac{1}{4} + \frac{3}{4}\left(\frac{1}{2}\right)^2$

4/ a) This is example 4.2.3 page 55 of reader

b) idem

c) This is proof of Theorem 4.4.5
for $f(l,n) = 1(l=n)$, $\delta(l,n) = 1(l=n)$

and using the rates of the time-reversed queue from ex 4.2.3

5/ correction: jobs served at queue 2 route to queue 3

a) $\bar{n} = (n_1, n_2, n_3)$ with $n_i$ = # customers in queue i

$S = \{\bar{n} : n_i \geq 0, i = 1,2,3\}$

$q(n, n+e_1) = \mu_0$

$q(n, n-e_1+e_2) = \frac{1}{3}\mu_1$

$q(n, n-e_1+e_3) = \frac{2}{3}\mu_1$

$q(n, n-e_2+e_3) = \mu_2$

$q(n, n-e_3) = \mu_3$

b/ traffic equations $\lambda_j = \mu_0 P_{0j} + \sum_{i=1}^{3} \lambda_i P_{ij}$ $j=1,2,3$

$\lambda_1 = \mu_0$
$\lambda_2 = \frac{1}{3}\lambda_1$
$\lambda_3 = \frac{2}{3}\lambda_1 + \lambda_2$

solution : $\lambda_1 = \mu_0$
$\lambda_2 = \frac{1}{3}\mu_0$
$\lambda_3 = \mu_0$ (obvious as all customers arrive at queue 3)

stability condition $\lambda_i < \mu_i$ $i=1,2,3$

so $\mu_0 < \mu_1$, $\mu_0 < 3\mu_2$, $\mu_0 < \mu_3$

stab. condition : $\mu_0 < \min\{\mu_1, 3\mu_2, \mu_3\}$

c/ $\pi(\bar{n}) = \prod_{i=1}^{3} (1-\rho_i)\rho_i^{n_i}$ $n_i \geq 0$ with $\rho_i = \lambda_i/\mu_i$

The network is a feed forward network so we can indeed prove this result using Burke's theorem

d/ Now $n_3 \leq N_3$ customers arriving to queue 3 will jump over queue 3 if $n_3 = N_3$

$\pi(\bar{n}) = (1-\rho_1)\rho_1^{n_1} (1-\rho_2)\rho_2^{n_2} \frac{1-\rho_3}{1-\rho_3^{N_3+1}} \rho_3^{n_3}$ $n_1 \geq 0$, $n_2 \geq 0$, $0 \leq n_3 \leq N_3$

proof: either use partial balance or Burke's theorem as the network remains feed forward and queue 3 is now a $M/M/1/N_3$ queue.