Exam Queueing Theory

Monday, May 21, 2012, 16.00–19.00.

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1. Jobs arrive at a single machine according to a Poisson process with a rate of 8 jobs per hour. The machine processes jobs in order of arrival at an exponential rate of 14 jobs per hour.

   a) Determine the mean production lead time (waiting time plus processing time) of a job.

   b) Determine the long-run fraction of jobs with a production lead time greater than 2 times the mean production lead time.

   It is possible to increase the production capacity by adding an extra, but old, machine. This machine processes jobs at an exponential rate of 2 jobs per hour. Now assume that both machines are being used, and that, when a job arrives at an empty system, this job will be send to the fast machine.

   c) The state of the system can be described by n, the number of jobs in the system; when there is only 1 job in the system, the machine processing the job has to be included in the state description, so $(1, f)$ (resp. $(1, s)$) is the state with 1 job on the fast (slow) machine. Show that

   $$p_0 = \frac{7}{27}, \quad p_{1,f} = \frac{3}{27}, \quad p_{1,s} = \frac{7}{27}, \quad p_n = \frac{5}{27}\left(\frac{1}{2}\right)^{n-2}, \quad n \geq 2.$$

   d) Determine the mean production lead time of a job. Is it a good decision to add the old machine?

2. Jobs arrive at a machine according to a Poisson process with a rate of 1 job per hour. The machine processes jobs in order of arrival and the processing times are exponential with a mean of 30 minutes. After processing a job, the machine has to be cleaned with probability $\frac{1}{8}$. The time to clean the machine is exponential with a mean of 1 hour.

   a) Show that the Laplace-Stieltjes transform of the waiting time in hours is given by

   $$\widetilde{W}(s) = \frac{3}{8} + \frac{9}{16} \cdot \frac{1}{1+2s} + \frac{1}{16} \cdot \frac{1}{1+\frac{2}{3}s}.$$

   b) Determine the long-run fraction of jobs for which the waiting time is longer than 2 hours.

   c) What is the mean sojourn time of a job?

3. Wafer pods (typically containing 25 wafers) have to be processed in an oxide furnace for the development of an oxide layer. The furnace processes pods in batches of two, at an exponential rate of 4 batches per hour. Interarrival times of pods are exponential with a mean of 20 minutes. Once two pods are collected (in a collector station just before the oxide furnace), the batch of two pods is released to the oxide furnace.

   a) What is the long-run fraction of time the furnace is processing batches?

b) Let a_n be the probability that there are n batches waiting or in process just before the arrival of a batch at the furnace. Show that

$$a_n = \frac{3}{4}\left(\frac{1}{4}\right)^n, \quad n = 0, 1, 2, \ldots$$

c) Determine the probability that the production lead time (waiting plus processing) of a batch is greater than two times the mean production lead time of a batch.

d) Determine the mean production lead time of a pod.

4. Packets arrive at a network interface according to a Poisson process with a rate of $\frac{1}{8}$ packets per time unit. Two types of packets can be distinguished: short packets, which are acknowledgements and constitute 40% of the incoming packets, and long data packets. The time to transmit a short packet is exactly 1 time unit, to transmit a long one takes exactly 10 time units. Packets are transmitted in order of arrival.

a) What is the mean waiting time of an arbitrary packet?

Short packets are given priority over long data packets. Transmission of packets may not be interrupted.

b) Determine the mean waiting time of a high-priority short packet and a low-priority long one.

The new IEEE 802.3az standard adopts the following procedure to save energy. When there are no packets, the interface turns into the low-energy sleep mode. The transition to sleep mode takes exactly 1 time unit. When the interface enters sleep mode, it will sleep until the first packet arrival. This is, of course, immediate when during the transition to sleep mode, already one or more packets arrived. The transition from sleep mode to active mode also takes exactly 1 time unit.

c) Determine the mean waiting time of a high-priority short packet and a low-priority long one under this energy-saving policy.

While the interface is asleep, the energy consumption is only 10% of the consumption under active mode; energy consumption during transitions is as under active mode.

d) What is the percentage of energy consumption saved under this energy-saving policy (in comparison to the always-active policy)?

Credits:

1a	b	c	d	2a	b	c	3a	b	c	d	4a	b	c	d
2	2	3	3	4	3	3	2	3	3	2	2	3	3	2