Exam Queueing Theory
Monday, June 16, 2014, 15.00 – 18.00 hour.

Credits:

1a	b	c	d	2a	b	c	3a	b	c	d	4a	b	c	d	e
3	3	2	2	4	3	3	2	3	2	3	2	2	2	2	2

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1. Jobs arrive at a machine according to a Poisson process with a rate of 9 jobs per hour. The machine processes the jobs at an exponential rate of 10 jobs per hour. Whenever the number of jobs in the system exceeds 2, an extra identical machine is immediately turned on and starts processing, and when the number of jobs gets down to 2 again, the machine just finishing a job, is turned off.

   a) Compute the limiting distribution of the number of jobs in the system.
   b) Compute the mean sojourn time $E(S)$ and compute the reduction of the mean sojourn time compared to the situation in which the second machine is never used.
   c) Compute the rate (average number of times per hour) at which an extra machine is turned on.
   d) What is the expected duration of a period during which both machines are simultaneously processing jobs?

2. At a loading/unloading station, trucks arrive according to a Poisson process. The mean time between two successive arrivals of trucks is 36 minutes. It turns out that 60% of the arriving trucks both have to be unloaded and loaded, 20% of the arriving trucks only have to be loaded and 20% of the arriving trucks only have to be unloaded. Both the loading times and the unloading times are exponentially distributed with a mean of 15 minutes. In the loading/unloading station one can only work on one truck simultaneously. The trucks are served in order of arrival.

   a) Show that the Laplace-Stieltjes transform in hours of the total service time (including both loading time and unloading time, if necessary) of a truck is given by
   $$\widetilde{B}(s) = \frac{80 + 8s}{5(4+s)^2}$$
   and the Laplace-Stieltjes transform in hours of the residual total service time by
   $$\widetilde{R}(s) = \frac{32 + 5s}{2(4+s)^2}.$$
   b) Show that the Laplace-Stieltjes transform of the waiting time in hours of a truck is given by
   $$\widetilde{W}(s) = \frac{1}{3} + \frac{9}{13} \cdot \frac{1}{1+s} - \frac{1}{39} \cdot \frac{16}{16+3s}.$$
   c) What is the mean number of trucks present at the loading/unloading station?

3. A server performs two types of tasks. On the one hand, he serves customers arriving according to a Poisson process with a rate of 10 customers per hour. The service times of these customers are Erlang-2 distributed with a mean of 4 minutes. On the other hand, he does administrative tasks, taking an exponentially distributed time with a mean of 5 minutes. Administrative tasks also arrive according to a Poisson process with a rate of 3 tasks per hour. Administrative tasks have a lower priority than the service of customers.

   a) Determine the mean sojourn time of customers in the case that administrative tasks may be interrupted.

   b) Determine the mean sojourn time of customers in the case that administrative tasks may *not* be interrupted.

   Now assume that administrative tasks are not arriving according to a Poisson process, but that they are always available (i.e., the server can always work on administrative tasks if he is not working on customers).

   c) Determine the mean sojourn time of customers in the case that administrative tasks may be interrupted.

   d) Determine the mean sojourn time of customers in the case that administrative tasks may *not* be interrupted.

4. Jobs arrive according to a Poisson stream with a rate of 1 job per hour at a manufacturing system consisting of a single machine. The mean processing time is 50 minutes, and the standard deviation is 25 minutes. The machine processes jobs in order of arrival.

   a) Calculate the mean waiting time.

   The power costs are 120 euros per hour to keep the machine on. To save power costs one decides to immediately switch off the machine as soon as the buffer is empty. While the machine is off it uses no power. The machine is switched on when a new job arrives. It takes an exponential setup time with a mean of 30 minutes before the machine can start processing jobs.

   b) Calculate the mean waiting time.

   c) What is the reduction in average power costs per hour?

   To reduce the impact on the waiting time, one decides, instead of immediately switching off the machine, to wait exactly 30 minutes. If after 30 minutes no job has arrived, the machine is switched off.

   d) Calculate the mean waiting time.

   e) What is now the reduction in average power costs per hour?