# Exam for LNMB/Mastermath Course on Scheduling

30 May 2016

This exam consists of:

- **8 pages.**
- **7 questions.**
- You can obtain a total of 100 points. Your exam grade will be the points you obtained divided by 10.

You are allowed to make use of one Din-A4 paper with handwritten notes (on both sides) and a non-programmable calculator.

When a proof is asked, please provide a mathematically sound proof, short but precise. Unless stated otherwise, you are always expected to (briefly) explain your answer.
In case the objective function is not explicitely specified, it is supposed to be a regular objective function, unless stated otherwise.

Please write your answers in the space provided for it, directly after the question. Do not forget to write your name, ID number and university on each page.

**Exercise 1** (15 points).
Consider the problem $1 \mid r_j \mid L_{\max}$. Show that this problem is strongly NP-hard by a reduction from 3-Partition. In 3-Partition, you are given $3m$ items with values $a_1, \ldots, a_{3m}$. The question is whether there exists a partition of $\{1, \ldots, 3m\}$ into $m$ disjoint sets $S_1, \ldots, S_m$ such that $|S_i| = 3$ and $\sum_{j \in S_i} a_j = B$ $(i = 1, \ldots, m)$, where $B = \frac{1}{m} \sum_{j=1}^{3m} a_j$.

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*Continue your answer of Exercise 1.*

**Exercise 2** (15 points).
Consider the problem $1\,|\,r_j, d_j = D\,|\,\sum_j U_j$. Develop a polynomial time algorithm that solves this problem to optimality.

**Exercise 3** (15 points).
Consider the problem $P2 \,|\,|\, L_{\max}$. Develop a Dynamic Programming algorithm that solves this problem to optimality.
What is the running time of your algorithm?

**Exercise 4** (15 points)**.**
Consider a single machine scheduling problem with release dates and preemption. Show that the SRPT rule at any time $t$ maximizes the number of completed jobs up to time $t$.

**Exercise 5** (15 points)**.**
The head-body-tail problem is the problem $1 \,|\, r_j, q_j \,|\, D_{\max}$, where $D_{\max}$ denotes the maximum delivery time. Show how this problem can be applied to find a lower bound for $\mathrm{J} \,|\,|\, C_{\max}$.

**Exercise 6** (15 points)**.**
Consider the problem $1 \,|\, r_j, pmtn \,|\, \sum_j w_j C_j$.

(a) The p-WSPT rule schedules at any moment in time among all jobs that are released and have not been completed the job with highest ratio $\frac{w_j}{p_j}$.

Show that this rule does not necessarily find an optimal solution.

(b) The p-WSRPT rule schedules at any moment in time, $t$, among all jobs that are released and have not been completed the job with highest ratio $\frac{w_j}{p_j(t)}$, where $p_j(t)$ denotes the remaining processing time of job $j$ at time $t$.

Show that this rule does not necessarily find an optimal solution.

**Exercise 7** (10 points).
Consider the stochastic scheduling problem $P2 \mid \mid \mathbb{E}[C_{\max}]$.
Show that the LEPT policy is not necessarily optimal.