Exam Mastermath course 'Scheduling' 2021 12-7-2021

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.

Important: Please write down the following **statement** on your first sheet of paper: "This exam will be solely undertaken by myself, without any assistance from others, and without use of sources other than my personal notes and the sources available on the Scheduling website of elo.mastermath.nl."

Note that you are only allowed to use your **personal notes** and the **material** that we provided on the Scheduling website of elo.masthermath.nl.

Next to this, when scanning your exam, you should place your **student ID** on the first page. The exam should be **handwritten**, either on paper or digital. Please start each question on a **new page**. At the end of the exam, you should scan your work and submit **one pdf file**.

Good luck!

Question 1 *(10 points)*:

Consider the following instance of problem $Q2||\sum C_j$:

$n = 6, p = (2, 3, a, 6, 8, 10), s = (1, b)$.

a) Give an optimal schedule with corresponding total completion time for $a = 5$ and $b = 2$. Explain in detail how you obtained this schedule. *(3 points)*
b) Indicate for which values of $(a, b) \in \mathbb{R}^2$ the found schedule stays optimal and express the resulting makespan in terms of a and b. Explain your result in detail. *(7 points)*

Question 2 *(10 points)*:

Consider the problem $1||\sum w_j U_j$. Even though this problem is NP-hard, it is possible to efficiently find the optimal solution when enough additional information is given.

a) Suppose it is known which jobs complete before their due dates and which do not. Explain how you can use this information to efficiently find an optimal solution. Prove your result. *(5 points)*
b) Suppose it is known that k jobs complete before their due dates and $k - 1$ of these jobs are given. Explain how you can use this information to efficiently find an optimal solution. Prove your result. *(5 points)*

Question 3 *(10 points)*:

Prove that problem $1|r_j|\sum w_j C_j$ is strongly NP-hard.

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Question 4 *(10 points)*:

Consider the problem $P||\sum C_j$.

a) Can you give a (tight) bound on the approximation ratio for the LPT rule? Prove your result. *(7 points)*
b) Can you give a (tight) bound on the approximation ratio for the LPT rule for instances with $p_{max} \leq c \cdot p_{min}$ and c a constant? Prove your result. *(3 points)*

Question 5 *(10 points, indication 300 words)*:

Describe the steps of the solution method used to solve the *OR rescheduling problem* and how this is used to develop the decision support system.

IMPORTANT: please submit **<u>one</u> pdf file**.

END OF THE EXAM