Exam Mastermath course 'Scheduling' 2021 — 21-6-2021

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.

Important: Please write down the following **statement** on your first sheet of paper: "This exam will be solely undertaken by myself, without any assistance from others, and without use of sources other than my personal notes and the sources available on the Scheduling website of elo.mastermath.nl."

Note that you are only allowed to use your **personal notes** and the **material** that we provided on the Scheduling website of elo.masthermath.nl.

Next to this, when scanning your exam, you should place your **student ID** on the first page. The exam should be **handwritten**, either on paper or digital. Please start each question on a **new page**. At the end of the exam, you should scan your work and submit **one pdf file**.

Good luck!

Question 1 *(10 points)*:

Consider the following instance of problem $F2||C_{max}$:

$n = 5, p = \begin{pmatrix} 4 & 4 & 3 & a & b \\ 5 & 3 & 4 & 2 & 4 \end{pmatrix}.$

a) Give the optimal schedule resulting from Johnson's algorithm with corresponding makespan for $a = 1$ and $b = 6$. Explain in detail how you obtained this schedule. *(3 points)*
b) Indicate for which values of $(a, b) \in \mathbb{R}^2$ the found schedule stays optimal and express the resulting makespan in terms of a and b. Explain your result in detail. *(7 points)*

Question 2 *(10 points)*:

Consider a single machine on which 2 jobs need to be scheduled. The processing times of the 2 jobs are distributed as follows:

$$P(P_1 = 1) = 3/4, \quad P(P_1 = 2) = 1/8, \quad P(P_1 = 3) = 1/8$$
$$P(P_2 = 1) = 1/2, \quad P(P_2 = 2) = 1/2$$

You are allowed to preempt the machine at discrete times $0,1,2, \ldots$. If job j is completed at time C_j, a reward $w_j\left(\frac{1}{2}\right)^{C_j}$ is received with $w_1 = 10$ and $w_2 = 5$. You want to maximize the total expected reward. Which job should start first in an optimal schedule? Should it ever be preempted? Explain your answer in detail.

Question 3 *(10 points)*:

Prove that problem $1|r_j|\sum U_j$ is strongly NP-hard.

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Question 4 *(10 points)*:

Consider the following algorithm for the head-body-tail problem $1|r_j, d_j < 0|L_{max}$.
First, apply the non-preemptive EDD rule. Now, let job c be a critical job, i.e., $L_c = L_{max}$, and let t be the earliest time such that the machine is not idle in the interval $[t, C_c]$. Denote the set of jobs processed in $[t, C_c]$ by Q. Now, if $d(Q) = \max_{j \in Q} d_j = d_c$, the EDD schedule is optimal. Otherwise, there exists an interference job b with $d_b > d_c$.
In the latter case, let $A = \{j: r_j \leq -d_j, j \neq b\}$, $B = \{j: r_j > -d_j, j \neq b\}$, and construct a second schedule by first scheduling the jobs in set A in order of non-decreasing release dates, then interference job b, and finally, the jobs in set B in order of non-decreasing due dates. Now, select the best schedule from the EDD schedule and this schedule.
For the given algorithm, provide an instance showing that the performance bound of $3/2$ is tight. Explain how you derived this instance.

Question 5 *(10 points, indication 300 words)*:

Describe the different steps of the planning process in railway scheduling and explain the dependencies between these steps.

IMPORTANT: please submit **one** **pdf file**.

END OF THE EXAM