Exam Mastermath course 'Scheduling' 2022 20-6-2022

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.

Note: You are only allowed to use the handout written by the lecturers. **Good luck!**

Question 1 *(10 points)*:

Consider the following instance of problem $J2||C_{\max}$:

$n = 7, p = \begin{pmatrix} 2 & 2 & 4 & 8 & 3 & 3 & - \\ 1 & 3 & 3 & 9 & 1 & 2 & 1 \end{pmatrix}$, jobs 2 and 5 first have to be processed on M_1, jobs 1, 3, 4, and 6 first have to be processed on M_2 and job 7 only has to be processed on M_2.

Apply the optimal algorithm presented in the lecture for problem $J2||C_{\max}$ to this instance. Explain the different steps, give the results of these steps and present the optimal solution.

Question 2 *(10 points)*:

Consider a machine environment α with job characteristics β. Suppose that we can find an optimal solution to $\alpha|\beta|\sum T_j$ in polynomial time. Use this to describe a polynomial time algorithm to solve $\alpha|\beta|L_{\max}$ and prove that this algorithm leads to an optimal solution.

Question 3 *(10 points)*:

Consider a special case of $1||\sum w_j U_j$ where the weight w_j of each job is the same as its processing time p_j and the due dates are all the same: $w_j = p_j$ and $d_j = d$ for all j. Provide a polynomial time algorithm for this special case or prove that this special case is NP-hard.

Question 4 *(10 points)*:

Consider problem $P||C_{\max}$ and consider only instances for which an optimal solution has at least 3 jobs on each machine. Prove that the LPT rule leads to a $\frac{4}{3} - \frac{1}{3m}$ approximation of the problem.

Question 5 *(10 points, indication 300 words)*:

Consider the Electric Vehicle charging problem with the additional option 'Vehicle-to-Grid'. Give a general description of this problem and explain how the 'Vehicle-to-grid' option changes the problem structure compared to the base problem without this option. Furthermore, give a sketch of how this problem can be solved.

END OF THE EXAM