Exam Mastermath course 'Scheduling' 2022 11-7-2022

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.

Note: You are only allowed to use the handout written by the lecturers. **Good luck!**

Question 1 *(10 points)*:

Consider the following instance of problem $1||\sum T_j$:

$n = 3, p = (4,6,9), d = (12,8,10)$.

Apply the optimal algorithm presented in the lecture for problem $1||\sum T_j$ to this instance. Explain the different steps, give the results of these steps and present the optimal solution.

Question 2 *(10 points)*:

For the following two problems, give an optimal scheduling rule (algorithm) for the problem and prove that it leads to an optimal solution.

a. Problem $1|r_j = r|L_{max}$. *(5 points)*
b. Problem $1|r_j|C_{max}$. *(5 points)*

Question 3 *(10 points)*:

Problem $O2|r_j|C_{max}$ is strongly NP-hard. Either give a proof that this problem is NP-hard or strongly NP-hard (you may choose this yourself).

Question 4 *(10 points)*:

EDD is a 2-approximation algorithm for the head-body-tail problem $1|r_j, d_j < 0|L_{max}$, i.e., $\frac{L_{max}(EDD)}{L^*_{max}} < 2$. Provide a family of two-job instances that shows that this performance bound is tight.

Hint: You may use that $L_{max}(EDD) - L^*_{max} < d_c$ and $L_{max}(EDD) - L^*_{max} < p_b$, where job c is a critical job, i.e., $L_c = L_{max}(EDD)$ and job b an interference job. An interference job is defined as follows. Let t be the earliest time in the EDD schedule for which the machine is not idle in the interval $[t, C_c]$. Denote the set of jobs processed in $[t, C_c]$ by Q. Now, there exists an interference job if there exists a job $b \in Q$ for which $d_b > d_c$.

Question 5 *(10 points, indication 300 words)*:

Describe the two different solution methods used to determine an operating room schedule which minimizes the number of required beds. Describe the differences and similarities between the two methods and the advantages and disadvantages of both methods.

END OF THE EXAM