Exam Mastermath course 'Scheduling' 2023   19-6-2023

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.

Note: You are only allowed to use the handout distributed by the lecturers. **Good luck!**

PLEASE USE A NEW SHEET OF PAPER FOR EACH QUESTION

Question 1 *(10 points)*:

Consider the following instance of problem $O2||C_{max}$:

$$n = 3, p = \begin{pmatrix} 2 & 1 & 4 \\ 2+r & 2 & 1 \end{pmatrix}$$

with $r \geq 0$.

Apply the optimal algorithm presented in the lecture to this instance. Explain the different steps, give the results of these steps and present the optimal solution including the makespan dependent on r.

Question 2 *(10 points)*:

Consider the problem $1|r_j, pmtn|\sum w_j C_j$.

a. The p-WSPT rule schedules at any moment in time among all released uncompleted jobs the job with the highest ratio $\frac{w_j}{p_j}$. Show that this rule does not necessarily find an optimal solution. *(5 points)*

b. The p-WSRPT rule schedules at any moment in time t among all released uncompleted jobs the job with the highest ratio $\frac{w_j}{p_j(t)}$, where $p_j(t)$ denotes the remaining processing time of job j at time t. Show that this rule does not necessarily find an optimal solution. *(5 points)*

Question 3 *(10 points)*:

Prove that problem $1|r_j|\sum T_j$ is strongly NP-hard.

Question 4 *(10 points)*:

Consider a single machine and three jobs. The distribution of the processing time of job j with $j = 1, 2, 3$, is discrete uniform over the set $\{10 - j, 10 - j + 1, \ldots, 10 + j\}$. We want to minimize $\mathbb{E}[\sum h(C_j)]$, where the function $h(C_j)$ is defined as follows.

$$h(C_j) = \begin{cases} 0, & C_j \leq 20 \\ C_j - 20, & C_j > 20 \end{cases}$$

a. Find the schedule(s) that minimize $\mathbb{E}[\sum h(C_j)]$ and compute the value of the objective function for the optimal schedule(s). *(7 points)*

b. Is the Largest Variance first (LV) rule or the Smallest Variance first (SV) rule optimal for this specific problem instance? Explain why. *(3 points)*

Question 5 *(10 points, indication 300 words)*:

Explain the core steps of the Triana concept for decentralized energy management and the way how profile steering can be used in this context.

END OF THE EXAM