Course 19.155120.0 “Scientific Computing” test T_3

June 1, 2012, 8:45–9:05

Your name: __________________________________

Your student number: __________________________________

Space for your drafts (will not be checked)

Question 1 (35 points) Sylvester equation $AX - XB = C$ is solved for given $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{k\times k}$, $C \in \mathbb{R}^{n\times k}$ and unknown $X \in \mathbb{R}^{n\times k}$.

Q1a (10 p) Specify $(\mathsf{vec}(X))^T$ in terms of its columns x_i, $i = 1, \dots, k$:
$(\mathsf{vec}(X))^T = \dots$

Q1b (10 p) The Sylvester equation is transformed into an equivalent linear system $\mathcal{A}\,\mathsf{vec}(X) = \mathsf{vec}(C)$. Specify, without proof, the missing terms in the following formula (here $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix):

$$\mathcal{A} = \ \dots\dots\dots \ - B^T \otimes I_n$$

Q1c (15 p) Write down the matrix $\mathcal{A}$ for A and B given below:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathcal{A} =$$

Question 2 (30 points) A nonlinear system of equation $F(x) = 0$ is solved by an inexact Newton method, where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth mapping.

Q2a (10 p) Complete the formula below for the matrix free multiplication of the Jacobian times a vector $w \in \mathbb{R}^n$ (here $\delta > 0$ is a small parameter and $x_c \in \mathbb{R}^n$ is the current solution vector):

$$F'(x_c)w \approx \frac{1}{\delta}(F(x_c + \delta w) - \ \dots\dots\dots\ ). \qquad (1)$$

Q2b (10 p) Estimate accuracy of approximation (1) above, i.e. prove that the approximation error is $\mathcal{O}(\delta^\ell)$ and specify ℓ.

Q2c (10 p) How could we improve the accuracy of approximation (1)?

Space for your drafts (will not be checked)

Question 3 (35 points) Consider the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update, for given $B_k \in \mathbb{R}^{n \times n}$ and vectors x_k, x_{k+1}, y_k:

$$B_{k+1} = B_k + \frac{y_k y_k^T}{y_k^T (x_{k+1} - x_k)} - \frac{B_k (x_{k+1} - x_k)(B_k (x_{k+1} - x_k))^T}{(x_{k+1} - x_k)^T B_k (x_{k+1} - x_k)}$$

Q3a (10 p) What is the rank of the matrix $y_k y_k^T$? Motivate your answer.

Q3b (10 p) What is the rank of the matrix $B_{k+1} - B_k$? Motivate your answer.

Q3c (15 p) Simplify
$B_{k+1}(x_{k+1} - x_k) = \ldots$