Time Series Analysis (& SI)—191571090

(Lecture notes are allowed)

$5x_2 - 20 - 5x_2$

$x_1 = 5 \Rightarrow -4(x_2 - 4) = -3$, $-4x_2 = -19$, $x_2 = 19/4$

Date: 31-10-2014
Place: CR-3H
Time: 08:45–11:45

$E(X_1) \cdot 11$

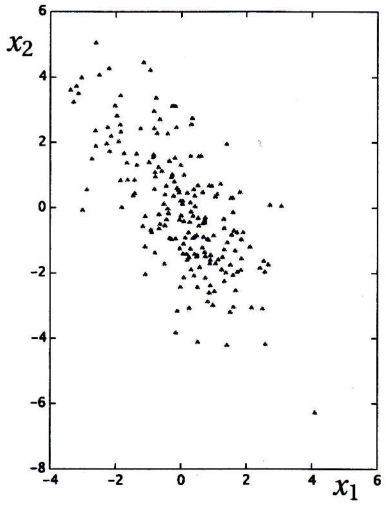
$(X_1 - 5)(X_2 - 4) = -3$

$X_1 \cdot X_2 - 4X_1 - 5X_2 + 20 = -3$

$X_1 \cdot X_2 - 4X_1 - 5X_2 = -23$

$X_1 = 2 \Rightarrow 2X_2 - 8 - 5X_2 = -23$, $-3X_2 = -15$, $X_2 = 5$

$X_1 = 8 \Rightarrow 8X_2 - 32 - 5X_2 = -23$, $3X_2 = 9$, $X_2 = 9/3$



1. Two questions:

   (a) Suppose $X = (X_1, X_2)$ has covariance matrix

   $$R_X = \begin{bmatrix} 9 & -3 \\ -3 & 1 \end{bmatrix}.$$

   $\sigma_{X_1} = 3$

   $\sigma_{X_2} = 1$

   Sketch a reasonable scatter plot of (X_1, X_2) of at least 20 points. (An example of a scatter plot is shown above; clearly this is not a reasonable one given our R_X.)

   (b) Let $X = \mathscr{H}(\epsilon)$ with ϵ_t white noise. If the system $\mathscr{H}$ is LTI and has "peak-to-peak gain" $\|h\|_1 \le 1$ then the "variance gain" $\operatorname{var}(X_t)/\operatorname{var}(\epsilon_t)$ is ≤ 1 as well?

2. Consider $X = \mathscr{H}(\epsilon)$ defined by the scheme

   $$X_t = aX_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}.$$

   (a) For which $a \in \mathbb{R}$ is this system asymptotically WSS?

   (b) Determine the impulse response h_t

   (c) Suppose the scheme is asymptotically WSS. Compute $\|h\|_1$.
   (The answer still depends on a.)

   (d) Is the scheme invertible?

   (e) Suppose the scheme is asymptotically WSS. Determine the TWO-step-ahead predictor scheme

3. Let X_t be the AR(2) process described by $(1-1.6q^{-1}+0.8q^{-2})X_t = \epsilon_t$ and suppose ϵ_t is white noise with variance 17/5.

   8 (a) Show that $r_X(0) = 45, r_X(1) = 40$ and $r_X(2) = 28$

   1 (b) Suppose we have $N = 100$ samples $X_1, \dots, X_N$ of this process and that we use it to fit an AR(2) scheme $(1-\hat{a}_1 q^{-1} - \hat{a}_2 q^{-2})X_t = \hat{\epsilon}_t$ using least-squares and where we assume that $\mathbb{E}\,X_t = 0$. Can you estimate $\mathrm{var}(\hat{a}_1)$ and $\mathrm{var}(\hat{a}_2)$?

8 4. Determine the coefficients h_j of the Savitsky-Golay filter

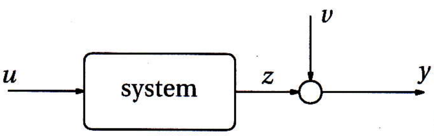
$$\hat{m}_0 = h_{-2}x_{t+2} + h_{-1}x_{t+1} + h_0 x_t + h_1 x_{t-1} + h_2 x_{t-2}$$

for degree 1 polynomial approximation (that is, in the lecture notes in Eqn. (6.8) we take the degree equal to 1.)

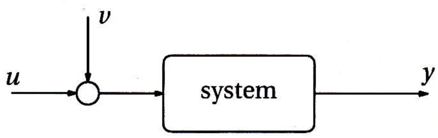
5. Let U_t and V_t be two zero mean WSS processes and suppose the two processes are uncorrelated. Assume further that the system $\mathcal{H}$ is LTI. In the notes it is proved that frequency response $\hat{h}(\omega)$ of the system equals

$$\hat{h}(\omega) = \frac{\phi_{yu}(\omega)}{\phi_u(\omega)}$$

for scheme (a) of the figure below. Does the result also hold for scheme (b) of the figure below? If so, show it. If not, derive a formula for $\hat{h}(\omega)$ in terms of $\phi_{yu}(\omega)$ abd $\phi_u(\omega)$.



(a)



(b)

6 12 6 4 4 = 32

problem:	1	2	3	4	5
points:	3+3	2+3+2+1+4	3+3	4	4

Exam grade is $1 + 9p/p_{\max}$.

$1 + \frac{9x}{32}$

$\frac{9x}{32} = 8.5$