Time Series Analysis (& SI)—191571090 – RETEST

(The first 1.5 hours it is CLOSED BOOK. The second 1.5 hours is OPEN BOOK.)

Date: 03-Feb-2017
Place: CR-2K
Time: 08:45–11:45

1. **CLOSED BOOK:** Three questions:

   (a) Prove that every covariance matrix R is positive semi-definite.
   ("Positive semi-definite" is also known as "nonnegative definite".)

   (b) What is the definition of the *coherence spectrum* of two jointly WSS processes U_t, Y_t?

   (c) Suppose ϵ_t is white. For which a, b with $|a| < 1$ is X_t defined as $X_t = \frac{1+b\mathrm{q}^{-1}}{1+a\mathrm{q}^{-1}}\epsilon_t$ asymptotically also white?

2. **CLOSED BOOK:** Consider the ARMA process

$$X_t = X_{t-1} - \frac{1}{4}X_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}$$

   with ϵ_t a zero-mean white noise process.

   (a) Is the scheme asymptotically wide sense stationary?

   (b) Is the scheme invertible?

   (c) Determine the 1-step ahead predictor

   (d) Determine the 2-step ahead predictor

   (e) Determine the spectral density function of X_t.

   (f) Is there a simpler scheme that defines the same asymptotically WSS process (in the sense of having the same mean and same covariance function)?

3. **CLOSED BOOK.** Suppose X_t is WSS and define

$$\hat{m}_N = \frac{1}{N}\sum_{t=0}^{N-1} X_t.$$

   Express the variance of $\hat{m}_N$ in terms of $r(k)$. (You have to *derive* the expression for this variance, not just recall it from memory.)

4. **CLOSED BOOK.** Mention at least one advantage of using windowing functions in the estimation of spectral densities.

5. **OPEN BOOK.** Let $X_t = A\cos(t+\phi)+\epsilon_t$ with $A>0$ and ϕ unknown constants. We want to estimate ϕ on the basis of $x_1,\ldots,x_N$ and we assume that ϵ_t is zero mean iid and normally distributed.

2 (a) Show that the joint probability density function of $X_1,\ldots,X_N$ is

$$\frac{1}{\sqrt{2\pi\sigma_\epsilon^{2N}}}e^{-\frac{1}{2\sigma_\epsilon^2}\sum_{t=1}^{N}(x_t-A\cos(t+\phi))^2}.$$

3 (b) Determine the Cramer-Rao lower bound for the estimation of ϕ. (You may want to use that $\cos(z)\sin(z)=\frac{1}{2}\sin(2z)$ and that $\cos^2(z)=\frac{1}{2}+\frac{1}{2}\cos(2z)$.)

2 (c) For large N the sum $\frac{1}{N}\sum_{t=1}^{N}\cos(2t+z)$ for every z is practically zero. Use this or your intuition to explain the influence of the value of the amplitude A on the quality of the estimate $\hat{\phi}$.

2 6. **OPEN BOOK.** Estimation of spectral densities is not trivial. An alternative would be to fit an AR-model first. Explain how you would use this to come up with an estimate of the spectral density function. (You may want to have a look at the explanation of the "cra" method in § 8.1.5 in Chapter 8.)

2 7. **OPEN BOOK:** Suppose X_t is an iid zero mean normally distributed white noise process. How large should N be so that $\hat{r}_N(0)$ has a standard deviation of about $0.1r(0)$?

2 8. **OPEN BOOK:** For which α is $U_t := (1+\alpha q^{-2})\epsilon_t$ not a sensible choice as input for system identification?

problem:	1	2	3	4	5	6	7	8
points:	2+2+2	2+2+2+2+2+2	3	2	2+3+2	2	2	2

Exam grade is $1+9p/p_{\max}$.