Time Series Analysis (& SI)—191571090

Date:	26-06-2020
Place:	At home!
Time:	08:45–11:45 (till 12:30 for students with special rights)
Course coordinator:	G. Meinsma
Allowed aids during test:	see below

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer! You are not allowed to use a calculator.

Integrity statement Please read the following paragraph carefully.

By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

The only allowed sources for this test are:

- the lecture notes "Time Series Analysis and System Identification" (pdf or printed)
- the slides (pdf or printed)
- electronic devices, but only to be used:
  - for downloading the test and afterwards uploading your work to Canvas
  - to show the test/book/slides on your screen
  - to write the test (in case you prefer to use a tablet instead of paper to write on)

problem:	1	2	3	4	5
points:	4	3+3+2	3+2+2+4	2+3	2+3+3

Exam grade is $1+9p/p_{\max}$.

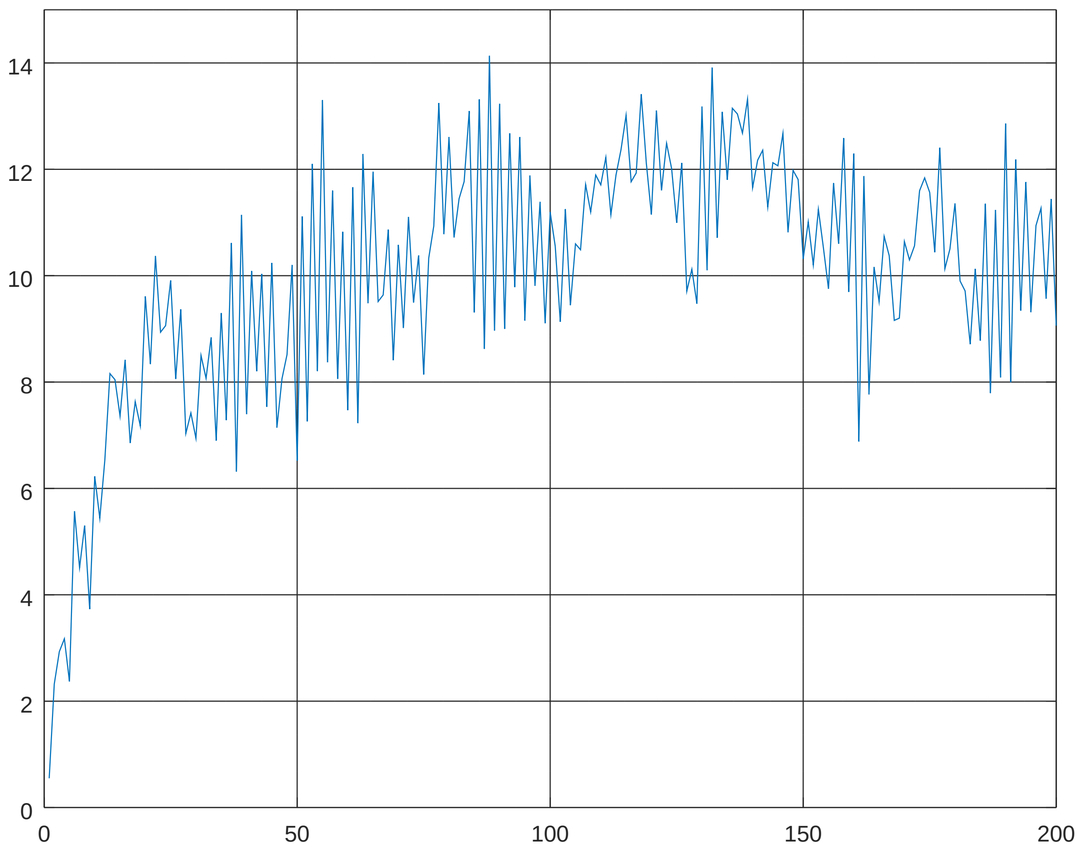
P.T.O.

A. **Copy the following text verbatim to the first page of your work (handwritten) and sign it. If you fail to do so, your test will not be graded:**

> *I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.*

B. **What programme do you follow (AM, AM+AP, AM+TCS, Minor, ...)**

C. **Are you entitled to extra time? (We will check this with CES.)**



1. The above is a realization of a second order process $(1 - a_2 \mathrm{q}^{-2})X_t = \epsilon_t$ in which $\mathbb{E}(\epsilon_t) = 2$. Determine reasonable values for a_2 and $\mathrm{var}(\epsilon_t)$.

2. Consider the linear process $X_t = \epsilon_t + 4\epsilon_{t-1} + 4\epsilon_{t-2}$, and assume ϵ_t has zero mean.

   (a) Describe this process as an *invertible* linear process $X_t = H(\mathrm{q})\eta_t$.

   (b) Determine the one-step ahead predictor $\hat{X}_{t|t-1}$.

   (c) Determine $\frac{1}{\pi}\int_0^{\pi} \phi_X(\omega)\,\mathrm{d}\mathrm{d}\omega$.

3. Let X_t be the AR(2) process described by $(1 - a_1 \mathrm{q}^{-1} - a_2 \mathrm{q}^{-2})X_t = \epsilon_t$ and whose covariance function $\gamma(k)$ satisfies $\gamma(0) = 6, \gamma(1) = 3, \gamma(2) = 0$.

   (a) Determine a_1, a_2

   (b) Calculate the variance σ_ϵ^2 of the white noise ϵ_t.

   (c) What is $\gamma(3)$?

   (d) Suppose we have $N = 102$ samples $X_1, ..., X_N$ of this process and that we use it to fit an AR(2) scheme $(1 - \hat{a}_1 \mathrm{q}^{-1} - \hat{a}_2 \mathrm{q}^{-2})X_t = \hat{\epsilon}_t$ using least-squares and where we assume that $\mathbb{E} X_t = 0$. Estimate $\mathrm{var}(\hat{a}_1)$ and $\mathrm{var}(\hat{a}_2)$

4. Suppose $X_0, \ldots, X_{N-1}$ are independent and that they are uniformly distributed on $[0, \theta]$ for some $\theta > 0$.

   (a) Show that the joint density function is $f(x_1, \ldots, x_N) = \begin{cases} \frac{1}{\theta^N} & \text{if all } x_i \in [0, \theta] \\ 0 & \text{elsewhere} \end{cases}$

   (b) Show that the maximum likelihood estimator of θ is $\hat{\theta} = \max_{t=0,\ldots,N-1} X_t$.

5. **System identification:** Consider

$$Y_t = \sum_{m=0}^{\infty} h_m U_{t-m} + V_t$$

   and suppose U_t is zero mean normally distributed iid white noise with variance 1. Assume, however, that V_t is some nonzero constant:

$$V_t = c \neq 0.$$

   This last assumption is uncommon (in Chapter 8 we normally assume that the mean of V_t is zero.) Based on many samples $U_0, \ldots, U_{N-1}, Y_0, \ldots, Y_{N-1}$ we do system identification:

$$\hat{h}_t := \frac{1}{N} \sum_{k=0}^{N-t-1} Y_{k+t} U_k, \qquad 0 \leq t \leq N-1.$$

   (a) Show that $h_t = \mathbb{E}(Y_{k+t} U_k)$ for all $t \geq 0$.

   (b) Suppose $h_t = 0$ for all $t \neq 0$ and that $c = 0$. Argue that $\mathrm{var}(\hat{h}_t) \approx 1/N$ if $N \gg t \gg 0$.

   (c) Suppose $h_t = 0$ for all $t \neq 0$ and that $c = 100$. What can you say about $\mathrm{var}(\hat{h}_t)$ if $N \gg t \gg 0$?