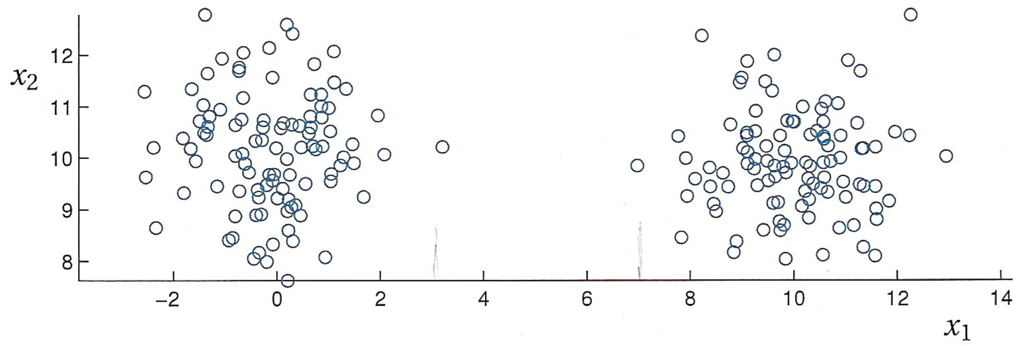
Time Series Analysis (& SI)—191571090

(Lecture notes are allowed)

Date: 30-01-2014
Place: RA-4237
Time: 13:45–16:45



1. Two questions:

   (a) Suppose we have many samples of $X = (X_1, X_2)$ as shown in the above figure. Make a rough guess of $\mathbb{E}(X_1)$ and $\mathbb{E}(X_2)$ and of the 2×2 covariance matrix R_X of X.

   (b) Suppose x_t is real valued. Show that its Fourier transform $\hat{x}(\omega)$ satisfies $|\hat{x}(\omega)| = |\hat{x}(-\omega)|$.

2. Let $a \in \mathbb{R}$. Consider the system

$$y_t = a y_{t-1} + u_t.$$

   (a) For which $a \in \mathbb{R}$ is the system asymptotically stable?

   (b) Let h be its impulse response. Is there an $a \in \mathbb{R}$ such that $\|h\|_1 = 1/4$?

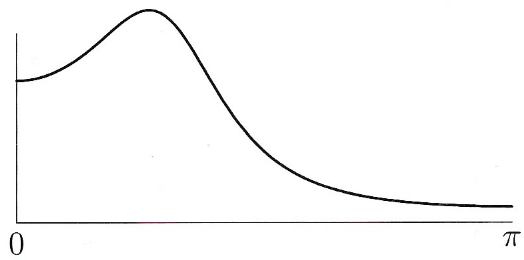
3. Let X_t be the AR(2) process described by $(1 - a_1 \mathrm{q}^{-1} - a_2 \mathrm{q}^{-2}) X_t = \epsilon_t$ and whose covariance function $r(k)$ satisfies $r(0) = 2, r(1) = 1, r(2) = 0$.

   (a) Determine a_1, a_2

   (b) Calculate the variance σ_ϵ^2 of the white noise ϵ.

   (c) What is $r(-3)$?

   (d) Determine $\phi(0)$. (In case you were not able to find $a_1, a_2, \sigma_\epsilon$ then you may want to determine a rough guess of $\phi(0)$ using that the spectral density $\phi(\omega)$ has the form as shown in the figure below.)



4. Consider the system

$$X_t = \frac{2}{3}X_{t-1} - \frac{1}{3}X_{t-2} + \epsilon_t$$

and assume that ϵ_t is zero mean white noise and that is has variance 1.

(a) Is X_t asymptotically wide-sense stationary?

(b) Determine the one-step-ahead predictor scheme

(c) Determine the mean square prediction error $\mathbb{E}(X_{t+2} - \hat{X}_{t+2|t})^2$ of the *two-step* ahead predictor $\hat{X}_{t+2|t}$.

5. Suppose $X_0, \dots, X_{N-1}$ are independent, identically distributed random variables with each X_i having probability density function $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-x^2/(2\sigma^2)}$ for some constant $\sigma > 0$.

(a) Determine the maximum likelihood estimator $\hat{\sigma}$ of σ, given $X_0, \dots, X_{N-1}$.

(b) Is the maximum likelihood estimator $\hat{\sigma}$ unbiased?

6. Suppose X_t is an independent, zero mean normally distributed white noise process. How long should we measure before the estimate $\operatorname{var}(\hat{r}_N(0))$ has a standard deviation of less than 0.01 of $r(0)$? [that is: what is the minimal N required?]

7. *System Identification.* Consider the standard system of system identification

$$y_t = \sum_{m=-\infty}^{\infty} h_m u_{t-m} + v_t.$$

What is a drawback of using $u_t = \cos(t) + \cos(2t)$ as input of an LTI system if we want to identify the system.

8. *System Identification method "cra".* Suppose $Y_t = \sum_{m=-\infty}^{\infty} h_m U_{t-m} + V_t$ and that U_t and V_t are WSS and that U_t, V_t is jointly WSS. The lecture notes claims that if we can find an $A(\mathrm{q})$ such that $\tilde{U}_t := A(\mathrm{q})U_t$ is white that then $h_t = r_{\tilde{y}\tilde{u}}(t)/r_{\tilde{u}}(0)$ where $\tilde{Y}_t = A(\mathrm{q})Y_t$. Explain why this is so.

problem:	1	2	3	4	5	6	7	8
points:	3+2	1+2	3+2+1+1	2+2+2	3+1	2	2	3

Exam grade is $1 + 9p/p_{\max}$.