UNIVERSITEIT TWENTE

Faculteit Elektrotechniek, Wiskunde en Informatica

Test exam 1, Random Signals and Filtering (201200135)

The solutions of the exercises need to be clearly formulated and written in a well-structured manner. Moreover, you always need to present a derivation or arguments to support your answer.

You can use one single-sided A4 page of handwritten notes with your exam.

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1. Consider $\Omega = \mathbb{R}$ and let $\mathcal{P}$ be such that

   $$\mathcal{P}([a,b])$$

   is a continuous function in a and b. Moreover, $\mathcal{P}$ satisfies all axioms of a probability measure. Prove that $\mathcal{P}(\{3\}) = 0$.

2. Consider the following linear system:

   $$\begin{aligned} X_{k+1} &= X_k + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

   where X_0, V_k and W_k are mutually independent and all have a uniform distribution on the interval $[0,1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine the density function associated to the stochastic variable X_1
   b) Determine $E[X_0|Y_0]$
   c) Determine $E[X_1|Y_0]$.

3. Consider the following linear system:

   $$\begin{aligned} X_{k+1} &= \tfrac{1}{2}X_k + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

   where X_0, V_k and W_k are mutually independent and all have a uniform distribution on the interval $[0,1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine $Z = E_{\text{lin}}[X_3|Y_0,Y_1,Y_2]$
   b) Determine the expectation of Z
   c) Determine the variance of Z.

4. Consider a stochastic vector X taking values in $\mathbb{R}^5$ with

   $$EX'X \le 10$$

   Argue how you would approach the question how many independent samples $\{x_1, \ldots x_N\}$ you would need to ensure that

   $$\mathcal{P}\left(\left| E[CX] - \frac{1}{N}\sum_{i=1}^{N} Cx_i \right| > 0.1 \right) < 0.05$$

   where

   $$C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$