Test Chapter 1. Linear Structures 1. 2015-201300056-1A: Structures and Models

25 September 2015, 13:45-15:15
This test consists of 6 questions.
Total: 30 points. Mark=[number of points]/3

1. [3pt] $V = \{(a_1, a_2, \ldots, a_n) : a_i \in \mathbb{Q},\ i = 1, 2, \ldots, n\}$. The addition and scalar multiplication are defined in a 'normal' way. Is V a vector space over $\mathbb{R}$?

2. W is a subset of matrices in $M_{n\times n}(\mathbb{R})$ where the sum of the entries in each row is equal to zero:

$$W = \{M \in M_{n\times n}(\mathbb{R}) : \sum_{j=1}^{n} M_{i,j} = 0, \quad i = 1, 2, \ldots, n\}.$$

(Here $M_{i,j}$ is the entry (i, j) of matrix M).

(a) [5pt] Show that W is a subspace of $M_{n\times n}(\mathbb{R})$.

(b) [2pt] What is the dimension of this subspace?

3. [5pt] V is a vector space, W is a subspace of V. Prove that if $S \subseteq W$, then $\mathrm{span}(S) \subseteq W$.

4. [5pt] Is the polynomial $x^3 + 11x^2 + 9x + 7$ a linear combination of the polynomials in $S = \{x^3 - x^2 + x - 1, x^3 + 2x^2 + 3x + 1, x^2 - 2x + 2\}$? If so, determine the corresponding coefficients. If not, why not?

5. [5pt] Check whether the set $S = \{(1, 1, 1), (-1, 0, 2), (2, 3, 4), (-1, 0, 0)\}$ contains a basis for $\mathbb{R}^3$. If so, give an example of a basis consisting of vectors in S. If not, why not?

6. [5pt] $S \subseteq V$ is a linearly independent set, $v \in \mathrm{span}(S)$. Prove that v can be *uniquely* written as

$$v = a_1u_1 + a_2u_2 + \cdots + a_nu_n,$$

where $u_1, u_2, \ldots, u_n \in S$ and $a_1, a_2, \ldots, a_n \in F$.