LS 1

23-10-2015

1 $T(f(t)) = (t-1) f''(t)$

a) $\beta = \{1, t, t^2, t^3\}$ $\quad \gamma = \{1, t, t^2\}$

$T(1) = 0 \qquad T(t) = 0 \qquad T(t^2) = 2t-2$

$T(t^3) = 6t^2 - 6t \qquad [T]_\beta^\gamma = \left([T(1)]_\gamma \; [T(t)]_\gamma \; [T(t^2)]_\gamma \; [T(t^3)]_\gamma \right)$

2pt

1pt

$$[T]_\beta^\gamma = \begin{pmatrix} 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$

3pt

b) $T(t^3 + 3t^2 + 3t + 1) = T((t+1)^3) = 6(t+1)(t-1)$

$= 6t^2 - 6$

1pt

$$[t^3 + 3t^3 + 3t + 1]_\beta = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \end{pmatrix}$$

1pt

$$[T]_\beta^\gamma \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \end{pmatrix} = \cancel{+6t + (6+6)t+} \begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix} =$$

$$= [6t^2 - 6]_\gamma$$

2pt

(2) (a) (i) T is ~~niet~~ one-to-one $\Rightarrow N(T) \neq \{0\}$ 1pt

$\Rightarrow \exists\, u \in N(T)$ zodat $u \neq \underline{0}$ 1pt

(ii) T is surjectief (onto) $\Rightarrow R(T) = W$

dim $W > 0 \Rightarrow \exists\, \omega \neq \underline{0}$, $\omega \in W$ 1pt

Omdat $R(T) = W$, er is $v \in V$ zodat

$T(v) = \omega \neq \underline{0} \Rightarrow v \in V$, $v \notin N(T)$ 1pt

(iii) Stel voor dat

$a \cdot v + a_1 u_1 + \ldots + a_k u_k = 0$ en niet alle coëfficiënten zijn gelijk aan 0.

$\{u_1 \ldots u_k\}$ - lin. onafh.

$\Rightarrow a_1 u_1 + \ldots + a_k u_k = 0 \Leftrightarrow a_1 = \ldots = a_k = 0.$

Het volgt dat $a \neq 0$ 1pt

Dan $v = -\frac{a_1}{a} u_1 - \frac{a_2}{a} u_2 - \ldots - \frac{a_k}{a} u_k$

$\Rightarrow v$ is een lin. comb. van $\{u_1, \ldots, u_k\} \subseteq N(T)$

$\Rightarrow v \in N(T)$ want $N(T)$ is een deelruimte.

Maar $v \notin N(T) \Rightarrow a = 0$

$= \{v_1, a_1, \ldots, a_k\}$ - lin. onafh. verzameling.

1pt.

(b) Dimensiestelling:

$$\dim(V) = \underbrace{\dim(N(T))}_{\substack{>0 \text{ want } T \\ \text{is niet one-to-one}}} + \underbrace{\dim(R(T))}_{= W \text{ (T is onto)}}$$ 0,5pt

0,5pt 0,5pt

$\Rightarrow \dim(V) > \dim(R(T)) = \dim(W)$ 0,5pt.

(c) $v \in N(T) \Rightarrow UT(v) = U(T(v)) = U(\underline{0}) = \underline{0}$

$\Rightarrow v \in N(UT) \Rightarrow N(T) \subseteq N(UT)$ 2pt

T ~~nicht~~ one-to-one $\Rightarrow N(T) \neq \{\underline{0}\}$ 1pt

$\Rightarrow N(UT) \neq \{\underline{0}\} \Rightarrow UT$ is nicht one-to-one 1pt

3

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 10 & 2 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 10 & 2 & 2 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 0 & -3 & 2 & 1 & -5 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 0 & +1 & -\frac{2}{3} & -\frac{1}{3} & \frac{5}{3} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{5}{3} & \frac{1}{3} & -\frac{5}{3} \\ 0 & 2 & 0 & \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{5}{3} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{5}{3} & \frac{1}{3} & -\frac{5}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{5}{3} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{6} & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{5}{3} \end{array}\right)$$

4pt

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1/2 & -2 \\ 1 & 1/2 & -1 \\ -2 & -1 & 5 \end{pmatrix}$$

2pt

(6) $Q = A$ - change of coordinate matrix ~~from~~ van β naar β'

$A^{-1} = Q^{-1}$ - change of coordinate matrix van β' naar β 2pt*

Of:

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}_{\beta}$ - coordinates in β

$$[x]_{\beta} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$[x]_{\beta'} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{zodat}$$

$$x_1 \underline{a}_1 + x_2 \underline{a}_2 + x_3 \underline{a}_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A[x]_{\beta'} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$[x]_{\beta'} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

2pt*

$$[x]_{\beta'} = \frac{1}{3} \begin{pmatrix} 2 & -\frac{1}{2} & -2 \\ 1 & \frac{1}{2} & -1 \\ -2 & -1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -5/3 \\ -1/3 \\ 11/3 \end{pmatrix}$$

2pt

* beide manieren zijn goed

Lin. systeem opstellen of Q^{-1} gebruiken - 2pt.

(a) $\underline{a}_2 = -2\underline{a}_1 = \begin{pmatrix} -2 \\ -4 \\ -2 \end{pmatrix}$ ~~1,5~~ 2

$\underline{a}_4 = \underline{a}_1 + 3\underline{a}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}$

~~1,5~~ 2

$A = \begin{pmatrix} 1 & -2 & -1 & -2 \\ 2 & -4 & 0 & 2 \\ 1 & -2 & 1 & -2 \end{pmatrix}$

(b) $K = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} t + \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} s \ : \ s,t \in \mathbb{R} \right\}$ 2pt

$x_4 = t$

$x_3 = 1 - 3t$

$x_2 = s$ 2pt

$x_1 = 1 - t + 2s$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} t$ 2pt

5 K_1 - oplossingsverzameling $Ax = b$

K_2 — " — $(CA)x = Cb$

$w \in K_1 \Rightarrow Aw = b$

$(CA)w = C(Aw) = Cb \Rightarrow w \in K_2$

$\Rightarrow K_1 \subseteq K_2$ 2pt

$w_2 \in K_2 \Rightarrow CAw = Cb$ C - inverteerbaar

$C^{-1}CAw = C^{-1}Cb \Rightarrow Aw = b \Rightarrow w \in K_1 \Rightarrow K_2 \subseteq K_1$

$\Rightarrow K_1 = K_2$ 1pt 3pt

(6)

$$\begin{vmatrix} -1-\lambda & 2 & 0 \\ -3 & 4-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\{-(1+\lambda)(4-\lambda)+6\}$$

2pt

1pt

$$= (1-\lambda)(\lambda^2 - 3\lambda + 2) = (1-\lambda)(\lambda-1)(\lambda-2)$$

$$\lambda = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \qquad = -(\lambda-1)^2(\lambda-2)$$

2pt

$\lambda = 1, \quad \lambda = 2$

Neem $\lambda = 1$

$$A - I \cdot 1 = \begin{pmatrix} -2 & 2 & 0 \\ -3 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1pt

$Ax = 0$

x_3 - vrij

x_2 - vrij 2pt

$x_1 = x_2$

Eén eigenvector: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

2pt