Test Chapter 1. Linear Structures 1. 2016-201300056-1A: Structures and Models

30 September 2015, 13:45-15:15
This test consists of 6 problems.
Total: 30 points. Grade=[number of points]/3

1. [5pt] $S = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$, and the addition and the scalar multiplication are defined as follows:

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2),$$
$$c(a_1, a_2) = (c|a_1|, ca_2).$$

Is S a vector space over $\mathbb{R}$? Why or why not?

2. S is a subset of a vector space V. Prove that $\mathrm{span}(S)$ is a subspace of V. o

3. [5pt] S_1 and S_2 are linear independent subsets of V. Prove that $S_1 \cap S_2$ is also a linear independent subset of V.

4. [5pt] Is the set of polynomials $S = \{x^3 + 2x, x^3 + x^2 + x, x^2 - x + 1, x^3 + 2x + 2\}$ a basis for $P_3(\mathbb{R})$?

5. [5pt] The set of matrices S is defined as follows:

$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}.$$

Can the matrix $\begin{pmatrix} 0 & 4 \\ 0 & 2 \end{pmatrix}$ be written as a linear combination of the matrices in S? If yes, find the coefficients. If not, why not?

6. [5pt] Let $\beta = \{u_1, u_2, \ldots, u_n\}$ be a subset of a vector space V. Furthermore, suppose that for any $v \in V$ there exist *unique* $a_1, a_2, \ldots, a_n$ such that

$$v = a_1u_1 + a_2u_2 + \cdots + a_nu_n.$$

Prove that β is a basis for V.