Test 2, Linear Structures II, 201300057

Date	:	December 23, 2016
Place	:	Therm
Time	:	13.45 – 15.15
Module-coordinator	:	B. Manthey
Instructor	:	H. Zwart

All answers must be motivated.
The use of (Scientific) calculator, formula sheet, or notes is not allowed.

1. Given the complex inner product space V. Let T be an operator from V to V.
   (a) Prove that if T is self-adjoint, then $\langle Tx, x\rangle$ is real for all $x \in V$.
   (b) Prove that if $\langle Tx, x\rangle$ is real for all $x \in V$, then T is self-adjoint.

2. Given the complex linear space V spanned by the functions: $\{e^{-x}, e^{-2x}\}$. On this space we have the inner product

$$\langle f, g\rangle = \int_0^\infty f(x)\overline{g(x)}dx. \tag{1}$$

   (a) Construct an orthonormal basis of V.
   (b) On the space V we consider the linear operator (the first derivative)

$$Tf = \frac{df}{dx}.$$

   Is T normal?

3. Label the following statements as true or false. If true, provide a proof, and when false provide a counter example. In all the items V is a complex inner product space.
   (a) If U_1 and U_2 are unitary operators on V, then U_2U_1 is also unitary.
   (b) The eigenvalues of a normal operator have modulus one.
   (c) If the self-adjoint operator T satisfies $T^3 = 0$, then $T = 0$.

Points [1]

Ex.1		Ex. 2		Ex. 3	
a	4	a	6	a	4
b	6	b	6	b	4
				c	6

[1]Total is 40. You will get 4 points for free