Partial Test 1, Lineaire Structures II, 201700140

Date : 1 december 2017

 $\begin{array}{lll} \text{Place} & : & \text{Therm} \\ \text{Time} & : & 13.45 - 15.15 \\ \text{Module-coordinator} & : & \text{J. de Jong} \\ \text{Instructor} & : & \text{H. Zwart} \\ \end{array}$

All answers must be motivated.

The use of (Scientific) calculator, formula sheet, or notes is not allowed.

1. Let $Q \in M_{n \times n}(\mathbb{C})$ be a given $n \times n$ matrix. With this Q we define the $2n \times 2n$ matrix

$$A = \left(egin{array}{cc} 0_n & -I_n \ Q & 0_n \end{array}
ight),$$

where 0_n is the $n \times n$ zero matrix, and I_n is the $n \times n$ identity matrix.

(a) Let $v \in \mathbb{C}^n$, $v \neq 0$ be such that $Qv = \lambda v$ for some $\lambda \in \mathbb{C}$. Determine all $\mu \in \mathbb{C}$ such that

$$w = \begin{pmatrix} v \\ \mu v \end{pmatrix}$$

is an eigenvector of A.

(b) Determine the eigenvalues and eigenvectors of

$$B = \left(\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right).$$

- (c) Is B diagonalizable?
- 2. Let \mathcal{P}_n be the set consisting of all $n \times n$ real matrices with non-negative coefficients. That is

$$\mathcal{P}_n = \{ A \in M_{n \times n}(\mathbb{R}) : A_{ij} \ge 0, i, j = 1, 2, \cdots, n \}.$$

- (a) Prove by induction that if $A \in \mathcal{P}_n$, then $A^k \in \mathcal{P}_n$ for all $k \in \mathbb{N}$.
- (b) Let $p(t) = (-1)^n (a_0 + a_1 t + \dots + a_{n-1} t^{n-1} + t^n)$ be the characteristic polynomial of A.

Show that if $A \in \mathcal{P}_n$ and not all $a_{\ell}, \ell = 1, \dots, n-1$ are zero, then there there exists a coefficient a_k that is negative, i.e, $a_k < 0$ for some $k \in \{1, \dots, n-1\}$.

3. Let $\mathcal E$ be the linear space consisting of all continuous functions from [-1,1] to $\mathbb R$ such that

$$f(x) = f(-x), \qquad x \in [-1, 1].$$

On this space we define the (candidate) inner product

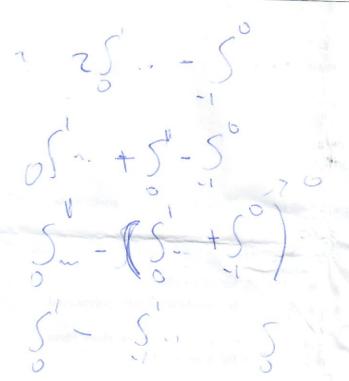
$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx.$$
 (1)

- (a) Prove that (1) defines an inner product on \mathcal{E} .
- (b) Construct a non-zero element of $\mathcal E$ that is orthogonal to f(x)=1.
- (c) Does the following define an inner product on \mathcal{E} ;

$$\langle f, g \rangle_b = 2 \int_0^1 f(x)g(x)dx - \int_{-1}^0 f(x)g(x)dx.$$
 (2)

Points ¹

Ex.1		Ex. 2		Ex. 3	
a	4	d	4	a	6
b	6	b	6	b	4
c	3			С	3



¹Total is 40. You will get 4 points for free