Partial Test 1, Lineaire Structures II, 201700140

Date	:	1 december 2017
Place	:	Therm
Time	:	13.45 – 15.15
Module-coordinator	:	J. de Jong
Instructor	:	H. Zwart

All answers must be motivated.
The use of (Scientific) calculator, formula sheet, or notes is not allowed.

1. Let $Q \in M_{n\times n}(\mathbb{C})$ be a given $n \times n$ matrix. With this Q we define the $2n \times 2n$ matrix

$$A = \begin{pmatrix} 0_n & -I_n \\ Q & 0_n \end{pmatrix},$$

where 0_n is the $n \times n$ zero matrix, and I_n is the $n \times n$ identity matrix.

   (a) Let $v \in \mathbb{C}^n$, $v \neq 0$ be such that $Qv = \lambda v$ for some $\lambda \in \mathbb{C}$. Determine all $\mu \in \mathbb{C}$ such that

$$w = \begin{pmatrix} v \\ \mu v \end{pmatrix}$$

   is an eigenvector of A.

   (b) Determine the eigenvalues and eigenvectors of

$$B = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}.$$

   (c) Is B diagonalizable?

2. Let $\mathcal{P}_n$ be the set consisting of all $n \times n$ real matrices with non-negative coefficients. That is

$$\mathcal{P}_n = \{A \in M_{n\times n}(\mathbb{R}) : A_{ij} \geq 0, i, j = 1, 2, \cdots, n\}.$$

   (a) Prove by induction that if $A \in \mathcal{P}_n$, then $A^k \in \mathcal{P}_n$ for all $k \in \mathbb{N}$.

   (b) Let $p(t) = (-1)^n \left(a_0 + a_1 t + \cdots + a_{n-1}t^{n-1} + t^n\right)$ be the characteristic polynomial of A.

   Show that if $A \in \mathcal{P}_n$ and not all $a_\ell, \ell = 1, \cdots, n-1$ are zero, then there there exists a coefficient a_k that is negative, i.e, $a_k < 0$ for some $k \in \{1, \cdots, n-1\}$.

3. Let $\mathcal{E}$ be the linear space consisting of all continuous functions from $[-1,1]$ to $\mathbb{R}$ such that

$$f(x) = f(-x), \qquad x \in [-1,1].$$

On this space we define the (candidate) inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx. \tag{1}$$

(a) Prove that (1) defines an inner product on $\mathcal{E}$.

(b) Construct a non-zero element of $\mathcal{E}$ that is orthogonal to $f(x) = 1$.

(c) Does the following define an inner product on $\mathcal{E}$;

$$\langle f, g \rangle_b = 2\int_0^1 f(x)g(x)dx - \int_{-1}^0 f(x)g(x)dx. \tag{2}$$

Points [1]

Ex.1		Ex. 2		Ex. 3	
a	4	a	4	a	6
b	6	b	6	b	4
c	3			c	3

$2\int_0^1 \cdots - \int_{-1}^0$

$\int_0^1 \cdots + \int_0^1 - \int_{-1}^0 > 0$

$\int_0^1 \cdots - \left(\int_0^1 \cdots + \int_{-1}^0\right)$

$\int_0^1 - \int_{-1}^1 \cdots \int_0$

[1] Total is 40. You will get 4 points for free