Partial Test 2, Lineaire Structures II, 201700140

Date	:	22 december 2017
Place	:	Sportcentrum
Time	:	13.45 – 15.15
Module-coordinator	:	J. de Jong
Instructor	:	H. Zwart

All answers must be motivated.
The use of (Scientific) calculator, formula sheet, or notes is not allowed.

1. Let Z be the linear space of 2 by 2 real matrices, with inner product

$$\left\langle \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \right\rangle = (A_{11}+A_{12})(B_{11}+B_{12}) + \\ 2(A_{12}+A_{21})(B_{12}+B_{21}) + 3A_{21}B_{21} + 4A_{22}B_{22}. \tag{1}$$

   (a) Define on Z the linear operator T as taking the transpose, i.e.,

   $$T(A) = A^t, \qquad A \in Z.$$

   Is T a self-adjoint operator on Z?

   Now we define the subspace W of Z as

   $$W = \{Q \in Z \mid Q_{21} = Q_{22} = 0\}.$$

   (b) Determine a basis of W that is orthonormal with respect to (1).

   (c) Given is $U = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right)$. Determine the best approximation of U in W, that is determine the orthogonal projection of U on W.

2. Let V be a real inner product space.

   (a) Show that the following relation holds between the inner product and its associated norm

   $$4\langle x, y\rangle = \|x+y\|^2 - \|x-y\|^2, \qquad \text{for all } x, y \in V.$$

   (b) Let T be a linear operator on V. Prove the following equivalence

   $$\|T(x)\| = \|x\| \quad \text{for all } x \in V$$
   $$\Leftrightarrow$$
   $$\langle T(x), T(y)\rangle = \langle x, y\rangle \quad \text{for all } x, y \in V$$

P.T.O.

3. Let Q be a linear operator on the finite-dimensional (complex) inner product space V. We assume that Q satisfies

$$Q^* = Q^2. \tag{2}$$

(a) Prove that Q is normal.

(b) Calculate all the (possible) complex eigenvalues of Q.

(c) Show that if Q is invertible, then $Q^3 = I$.

Points [1]

Ex.1		Ex. 2		Ex. 3	
a	4	a	4	a	3
b	5	b	6	b	6
c	5			c	3

[1] Total is 40. You will get 4 points for free