Test Linear Structures 2.
Applied Mathematics, 2023-1B: Structures and Systems

This exam consists of 10 problems which are divided into two parts:
Grasple (digital): 8 problems
Open Questions (written): 2 problems.

Grasple (This morning)
Enter your answers in Grasple in the required form. Follow the instructions precisely.
For the statements, you choose one of three options: true (T), false (F), or no answer (N).
For each correct T or F you will receive (partial) points.
One incorrect T or F results in zero points for that entire question.
Total score for Grasple: 40 points.
Required score: 20 points.

Open Questions (Now)
Write the solutions following the four steps.
Step 1. State the important information and summarize the problem.
Step 2. Devise a plan.
Step 3. Execute the plan.
Step 4. Analyze your solution and the answer.
Think, for example, of the following questions:
– Can you interpret the solution and/or the result intuitively?
– Does the solution/result make sense in special cases?
– Which role each condition played in the solution?
– Could you relax some assumptions of the problem?
– Is the problem related to other problems or results?
Total score for Open Questions: 40 points.
Steps 1+2: 40%, Step 3: 40%, Step 4: 20%.
Required score: 20 points.
Grade: 1+9(number of points)/80.

A (graphical) calculator is not needed and is **not allowed** at the exam.

PART 2: Open questions.

For each exercise, you can combine parts a) and b) in steps 1 and 4. That is, you only need to write 1 problem statement and 1 reflection for exercise 9. You also need only one of each for exercise 10.

[20pt] Consider a square matrix A. Let $B = P^{-1}AP$ for some invertible matrix P.

1. Prove that for any eigenvector $\boldsymbol{v}$ of A: $P^{-1}\boldsymbol{v}$ is an eigenvector of B.
2. Prove that if A is diagonalizable, then B is diagonalizable.

[20pt] Consider finite-dimensional real inner product space V.
For each of the following statements, either prove it, or give a counterexample. If you give a counterexample, then also prove that it is a counterexample.

1. For any self-adjoint linear operator T on V: There exists a self-adjoint operator U on V for which $U^2 = T$.
2. For any self-adjoint linear operator T on V: There exists a self-adjoint operator U on V for which $U^3 = T$.