# Solution/Correction standard, Test Mathematics A; September 23, 2016.

1. $A_1 = [-1, 5)$, $A_2 = [2, 10)$, $A_3 = [-3, 15)$, $A_4 = [4, 20)$. So

$$\bigcap_{k=1}^{4} A_k = [4, 5) \quad \text{and}$$

$$\bigcup_{k=1}^{4} A_k = [-3, 20)$$

2. If $x \in \mathbb{N}$, $y \in \mathbb{N}$ then $\exists x \forall y\, (x \leq y)$ is true; take $x = 1$.

   If $x \in \mathbb{N}$, $y \in \mathbb{N}$ then $\forall x \exists y\, (x \leq y)$ is true; for any $x$ take $y = x$.

   If $x \in \mathbb{Z}$, $y \in \mathbb{Z}$ then $\exists x \forall y\, (x \leq y)$ is false.
   If such an $x$ would exist then, if we choose $y = x - 1$, we get $x > y$.

   If $x \in \mathbb{Z}$, $y \in \mathbb{Z}$ then $\forall x \exists y\, (x \leq y)$ is true; for any $x$ take $y = x$.

   If no argumentation is provided: at most 1 pt for the entire exercise.

3. (a) Proof by contradiction. Suppose that $k$ is both even and odd.
   Then there exist $m, n \in \mathbb{Z}$ such that $k = 2m$ and $k = 2n + 1$. Then $2m = 2n + 1$. But then $2(m - n) = 1$ and so $m - n = \frac{1}{2}$, contradicting $m, n \in \mathbb{Z}$. So $k$ cannot be both even and odd.

   (b) Basis step for $n = 1$:

$$\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{2} \quad \text{and also} \quad \frac{1}{1+1} = \frac{1}{2}.$$

   So the statement is correct for $n = 1$. Induction step:
   Let $k \geq 1$ and suppose that:

$$\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1} \qquad \text{(Induction hypothesis: IH)}$$

   We must show that IH implies:

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{(k+1)+1} = \frac{k+1}{k+2}.$$

   Well:

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}.$$

where the second equality follows from IH.

(From the proof it must be crystal clear whàt is supposed (1 pt) and whàt must be proved (0.5 pt). In case of nonsense formulations like “Suppose it is correct FOR ALL $n$, so it also holds for $n+1$”: at most 1 pt for the entire exercise)

4. (a) $10$ choices for each letter. So $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$ possible selections.

   (b) $10$ choices for the first letter, $9$ for the second, $8$ for the third and $7$ for the fourth.
   So $10 \cdot 9 \cdot 8 \cdot 7 = 5,040$ possible selections.

   (c) The number of selections of $4$ elements from a set of $10$ elements.
   So $\binom{10}{4} = 210$ possible selections.

Some argumentation/calculation is necessary; just the correct answer: 0.5 pt per item.