

Course : Mathematics β 1, Bernoulli

Date : January 8, 2018 Time : 18:15h – 21:15h

Motivate all your answers.

The use of electronic devices is not allowed.

1. [3 pt] Solve the initial value problem

$$\begin{cases} y' + \frac{1}{(x+1)}y = \frac{x}{(x+1)}, & x > -1, \\ y(0) = 1. \end{cases}$$

2. Define
$$z = \frac{4i}{1 + i\sqrt{3}}$$
.

(a) [2 pt] Find the modulus (absolute value) and the argument of z.

(b) [2 pt] Find the real and imaginary part of z^3 .

(c) [2 pt] Find the three cube roots of 8i.

3. (a) [4 pt] Solve y(t) from the following initial value problem

$$\begin{cases} y'' + 6y' + 9y = 0, \\ y(0) = 1, \\ y'(0) = 0. \end{cases}$$

(b) [3 pt] 5 Write the real general solution of the second order differential equation

$$y'' + 6y' + 9y = 3$$

in set notation.

4. Let
$$\mathbf{u} = (\frac{3}{5}, \frac{4}{5}, 0)$$
, $\mathbf{v} = (\frac{4}{5}, \frac{-3}{5}, 0)$ and $\mathbf{w} = (0, 0, 1)$.

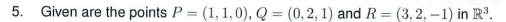
(a) [2 pt] Show that

$$|\mathbf{u}| = |\mathbf{v}| = |\mathbf{w}| = 1;$$

 $\mathbf{u} \perp \mathbf{v}$;

 $u \perp w$ and $v \perp w$.

(b) [3 pt] \circ Let $\mathbf{r} = \alpha \mathbf{u} + \beta \mathbf{v} + \gamma \mathbf{w}$, with $\alpha, \beta, \gamma \in \mathbb{R}$. Show that $\alpha = \mathbf{r} \bullet \mathbf{u}$, $\beta = \mathbf{r} \bullet \mathbf{v}$ and $\gamma = \mathbf{r} \bullet \mathbf{w}$.



- (a) [2 pt] Determine the vector equation of the line ℓ going through P and Q.
- (b) [2 pt] Find an equation of the plane that passes through the three points P, Q and R.
- 6. Consider the function $f: \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq \pm 1\\ \frac{1}{2} & \text{if } x = \pm 1. \end{cases}$$

- (a) [3 pt] Find all $x \in \mathbb{R}$ in which f is continuous.
- (b) [3 pt] Are there extreme values of f on \mathbb{R} ? Motivate your answer.
- 7. (a) [2 pt] Formulate the Mean Value Theorem.
 - (b) [3 pt] Let I be an open interval in \mathbb{R} . Suppose that f is a function on I that is differentiable. Give a proof for the following statement:

If f'(x) > 0 for all x in I, then f is increasing on I.

Total: 36 points

