[redacted]

Course : **Mathematics β1, Bernoulli**

Date : January 8, 2018
Time : 18:15h – 21:15h

Motivate all your answers.
The use of electronic devices is not allowed.

1. [3 pt] Solve the initial value problem

$$\begin{cases} y' + \dfrac{1}{(x+1)}y = \dfrac{x}{(x+1)}, & x > -1, \\ y(0) = 1. \end{cases}$$

2. Define $z = \dfrac{4i}{1+i\sqrt{3}}$.

(a) [2 pt] Find the modulus (absolute value) and the argument of z.

(b) [2 pt] Find the real and imaginary part of z^3.

(c) [2 pt] Find the three cube roots of $8i$.

3. (a) [4 pt] Solve $y(t)$ from the following initial value problem

$$\begin{cases} y'' + 6y' + 9y = 0, \\ y(0) = 1, \\ y'(0) = 0. \end{cases}$$

(b) [3 pt] Write the real general solution of the second order differential equation

$$y'' + 6y' + 9y = 3$$

in set notation.

4. Let $\mathbf{u} = (\frac{3}{5}, \frac{4}{5}, 0)$, $\mathbf{v} = (\frac{4}{5}, \frac{-3}{5}, 0)$ and $\mathbf{w} = (0, 0, 1)$.

(a) [2 pt] Show that

$|\mathbf{u}| = |\mathbf{v}| = |\mathbf{w}| = 1;$
$\mathbf{u} \perp \mathbf{v};$
$\mathbf{u} \perp \mathbf{w}$ and $\mathbf{v} \perp \mathbf{w}$.

(b) [3 pt] Let $\mathbf{r} = \alpha\mathbf{u} + \beta\mathbf{v} + \gamma\mathbf{w}$, with $\alpha,\ \beta,\ \gamma \in \mathbb{R}$. Show that $\alpha = \mathbf{r} \bullet \mathbf{u}$, $\beta = \mathbf{r} \bullet \mathbf{v}$ and $\gamma = \mathbf{r} \bullet \mathbf{w}$.

5. Given are the points $P = (1, 1, 0)$, $Q = (0, 2, 1)$ and $R = (3, 2, -1)$ in $\mathbb{R}^3$.

(a) [2 pt] Determine the vector equation of the line ℓ going through P and Q.

(b) [2 pt] Find an equation of the plane that passes through the three points P, Q and R.

6. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq \pm 1 \\[2ex] \frac{1}{2} & \text{if } x = \pm 1. \end{cases}$$

(a) [3 pt] Find all $x \in \mathbb{R}$ in which f is continuous.

(b) [3 pt] Are there extreme values of f on $\mathbb{R}$? Motivate your answer.

7. (a) [2 pt] Formulate the Mean Value Theorem.

(b) [3 pt] Let I be an open interval in $\mathbb{R}$. Suppose that f is a function on I that is differentiable. Give a proof for the following statement:

If $f'(x) > 0$ for all x in I, then f is increasing on I.

[redacted]

Total: 36 points