Course : **Calculus 1 for AM and TN**

Date : October 26, 2018
Time : 13.45 – 16.45 hrs

All answers must be motivated and clearly formulated.
The use of a calculator is not allowed.
This exam paper has eight (8) questions.

1. [4pt]

   a) [2pt] Describe (in words or with a clear picture) the region or object in $\mathbb{R}^3$ given by $\{x = (x, y, z) \in \mathbb{R}^3 | \, x \leq 3, z \geq 0\}$. Replace with W to avoid confusion

   b) [2pt] Give the vector parametric equation of the line that passes through the point $(1, 2, 3)$ and is orthogonal to the plane $2x - 3y + z = 5$.

2. [4pt] You are given the vectors $\mathbf{u} =< 1, 2, -2 >$ and $\mathbf{v} =< 3, 0, -4 >$.

   a) [2pt] Find the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$.

   b) [1pt] Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal? Explain your answer.

   c) [1pt] Find a vector of length 5 in the direction of $\mathbf{u}$.

3. [10pt] Solve the initial-value problems or differential equations below.

   a) [4pt]

$$\begin{cases} y'' + 6y' + 5y = 0, \\ y(0) = 0, \\ y'(0) = 3. \end{cases}$$

   b) [3pt]

$$xy' + 3y = \frac{\sin x}{x^2}, \quad x > 0$$

   c) [3pt]

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{e^{(2x-y)}}{e^{(x+y)}}, \\ y(0) = 1 \end{cases}$$

4. [8pt]

a) [2pt] Formulate (but do not prove) de Moivre's Theorem for complex numbers.

b) [2pt] You are given the complex numbers $z = 3 - i$ and $w = 2 + 2i$. Give the following in Cartesian (rectangular) form:

i) $z + w$

ii) $\frac{z}{w}$

c) [4pt] Solve for all z such that $z^3 = i$. List all solutions in polar form and plot all solutions on the complex plane.

5. [3pt] $f(x)$ is said to be differentiable at $x = a$ if $f'(a)$ exists. Use the definition of the derivative to prove that differentiability implies continuity.

6. [2pt] Use the Squeeze Theorem to prove that

$$\lim_{\theta \to 0} \theta^2 \sin\left(\frac{1}{\theta}\right) = 0$$

7. [3pt] Consider the function $f(x) = \cos(x + \sin(x))$ on the interval $[0, \pi]$. On the grounds of what theorem can we be sure that the function has a zero within the given interval? Explain your reasoning and formulate the theorem that you have used.

8. [2pt] Use L'Hospitals rule to evaluate the limit

$$\lim_{x \to 0} \frac{(x + 1 - e^x)}{x^2}.$$

Include in your answer an explanation of why L'Hospital's rule is applicable in this case.

Total: 36 points