Course	:	Mathematics β II
Date	:	January 8th 2016
Time	:	13:45-15:45

Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. Let

$$f(x) = \begin{cases} x^2 \cos(\frac{1}{x}) & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$

   (a) Determine $f'(x)$ for $x \neq 0$.

   (b) Use the definition of derivative to obtain that $f'(0) = 0$.

   (c) Calculate $f'(\frac{2}{(2k+1)\pi})$, $k \in \mathbb{N}$.

   (d) Is $f'(x)$ continuous in 0?

2. Calculate

$$\lim_{x \to 0} \frac{e^{2x} - 1}{xe^{2x}}.$$

3. Calculate, if the limit exists

$$\lim_{(x,y) \to (0,0)} \frac{x^2 y^2}{2x^4 + y^4}.$$

4. Let S_n be given by

$$S_n = \sum_{k=1}^{n} \frac{2k-1}{n^2} \frac{k}{n}.$$

   We want to interpret S_n as a Riemann sum of the function $f(x) = \sqrt{x}$ on the interval $[0,1]$ with $\Delta_k = \dfrac{2k-1}{n^2}$.

   (a) For $n = 5$ depict the corresponding partition of $[0,1]$ and determine $x_0, x_1, x_2, x_3, x_4, x_5$.

   (b) For general n determine the corresponding partition $P_n = \{x_0, x_1, \ldots, x_{n-1}, x_n\}$ of the interval $[0,1]$.

   (c) Calculate

$$\lim_{n \to \infty} S_n.$$

5. (a) Formulate the Mean Value Theorem for integrals.

   (b) Determine the average value of $f(x) = x^7 e^{x^4}$ on the interval $[0, 2]$.

6. Let $f(x) = e^x \sin(x)$.

   (a) Give the Taylor Series expansion of $f(x)$ about 0 up to and including the term of degree four.

   (b) Determine the radius of convergence of the Taylor series of $f(x)$ about 0.

Points: **Ex 1**, a: 2, b: 2, c: 1, d: 2, **Ex 2**: 5, **Ex 3**: 4, **Ex 4**: a: 3, b: 2, c: 3, **Ex 5**: a: 2, b: 4, **Ex 6**: a: 4, b: 2.