Math β2 8-1-2016

1 (a) $f'(x) = 2x\cos\frac{1}{x} + (x^2\sin\frac{1}{x})\cdot\frac{1}{x^2}$

$= 2x\cos\frac{1}{x} + \sin\frac{1}{x}$

(b) $\lim\limits_{h\to 0}\frac{f(h)-f(0)}{h} = \lim\limits_{h\to 0}\frac{h^2\cos\frac{1}{h}}{h} = \lim\limits_{h\to 0} \underbrace{h}_{\to 0}\underbrace{\cos\frac{1}{h}}_{\text{bounded}} = 0.$

$\Rightarrow f'(0) = 0.$

(c) $f'\left(\frac{2}{(2k+1)\pi}\right) = \cancel{\frac{4}{(2k+1)^2\pi^2}\cos((2k+1)\pi)}$

$\underbrace{\frac{4}{(2k+1)\pi}\cos((2k+1)\pi)}_{\to 0\ (k\to\infty)} + \underbrace{\sin\left(\frac{(2k+1)}{2}\pi\right)}_{=1} \to 1 \quad (k\to\infty)$

(d) No!

$\lim\limits_{k\to\infty} f'\left(\frac{2}{(2k+1)\pi}\right) = 1 \neq 0$, $\frac{2}{(2k+1)\pi}\to 0.$

for continuity of $f'(x)$ in 0 we should have $\lim\limits_{k\to\infty} a_k = 0 \Rightarrow \lim\limits_{k\to\infty} f'(a_k) = f'(0) = 0.$

2. $\lim_{x \to 0} \frac{e^{2x}-1}{xe^{2x}} = 2$, because by l'Hôpital:

$$\lim_{x \to 0} \frac{2e^{x}}{e^{2x}+2xe^{2x}} = 2$$

3. $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{2x^4+y^4}$ does not exist because:

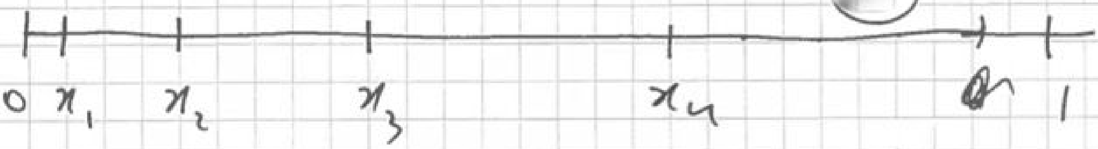
$$\lim_{x \to 0} \frac{x^4}{2x^4+x^4} = \frac{1}{3}$$ (along $x=y$).

(bijvoorbeeld)

$$\lim_{x\to 0, y=0} \frac{x^2y^2}{2x^4+y^4} = 0 \neq \frac{1}{3}.$$

4. a $n=5$: $\underbrace{x_0=0 \quad \Delta_1=\frac{1}{25} \quad \Delta_2=\frac{3}{25} \quad \Delta_3=\frac{5}{25} \quad \Delta_4=\frac{7}{25} \quad \Delta_5=\frac{9}{25}}$

$$\left.\begin{aligned}
x_1 &= x_0+\Delta_1 = \frac{1}{25}\\
x_2 &= x_1+\Delta_2 = \frac{4}{25}\\
x_3 &= x_2+\Delta_3 = \frac{9}{25}\\
x_4 &= x_3+\Delta_4 = \frac{16}{25}\\
x_5 &= x_4+\Delta_5 = \frac{25}{25}=1.
\end{aligned}\right\}$$



b. $x_0=0 \quad \Delta_1=\frac{1}{n^2} \quad \Delta_2=\frac{3}{n^2}, \quad \Delta_3=\frac{5}{n^2}, \to \quad \Delta_n=\frac{2n-1}{n^2}$

$$\left\{\begin{aligned}
x_1 &= \frac{1}{n^2}\\
x_2 &= \frac{4}{n^2}\\
x_3 &= \frac{9}{n^2}\\
&\vdots\\
x_{n-1} &= \frac{(n-1)^2}{n^2}\\
x_n &= 1
\end{aligned}\right.$$

(c) $$S_n = \sum_{k=1}^{n} \Delta_k \sqrt{x_k}$$

$$\left.\begin{aligned}
&= \sum_{k=1}^{n} \frac{2k-1}{n^2}\cdot\sqrt{\frac{k^2}{n^2}}\\
&= \sum_{k=1}^{n} \frac{2k-1}{n^2}\cdot\frac{k}{n}
\end{aligned}\right\}$$

$$\xrightarrow{k\to\infty} \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \cancel{}\ 2/3$$

5. (a) $f(x)$ continuous on $[a,b]$:

$\exists\, c \in [a,b]$ s.t.

$$\int_a^b f(x)\,dx = (b-a)f(c)$$

(b) $$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_0^2 x^7 e^{x^4}\,dx$$

$u = x^4$

$du = 4x^3\,dx$

$$= \frac{1}{8}\int_0^{16} u e^u\,du = \frac{1}{8} u e^u \Big|_0^{16} - \frac{1}{8}\int_0^{16} e^u\,du$$

$$= \frac{1}{8}\left(16e^{16} - e^{16} + 1\right) = \frac{1}{8}\left(15e^{16} + 1\right).$$

6. (a) $$f(x) = e^x \sin x = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \ldots\right)\left(x - \frac{x^3}{6} + \ldots\right)$$

$$= x + x^2 + \left(-\frac{x^3}{6} + \frac{1}{2}x^3\right) + \frac{1}{6}x^4 - \frac{x^4}{6} + \ldots$$

$$= x + x^2 + \frac{1}{3}x^3 \ldots$$

(b) $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ and $\sin x = \sum_{l=0}^{\infty} (-1)^l \frac{x^{2l+1}}{(2l+1)!}$

both have $R = \infty$.

$\Rightarrow$ so does $f(x)$.