Course : Mathematics β II
Date : January 11th 2018
Time : 13:45-15:45

Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. Let S_n be given by

$$S_n = \sum_{k=1}^{n} \frac{n}{n^2 + k^2}.$$

   (a) Interpret S_n as a Riemann sum of a function $f(x)$ on the interval $[0, 1]$. Hint: take the partition $P_n = \{0, 1/n, 2/n, \ldots, (n-1)/n, 1\}$ as the starting point for rewriting S_n as Riemann sum and determine the function $f(x)$.

   (b) Now calculate

$$\lim_{n\to\infty} S_n.$$

2. (a) Formulate the Mean Value Theorem for Integrals.

   (b) Calculate the average value of $f(x) = x^2 \ln(x^4)$ on the interval $[2, 5]$.

3. The sphere S defined by by

$$x^2 + y^2 + z^2 = 1$$

and the elliptic cylinder C defined by

$$x^2 + 4y^2 = 1$$

intersect in two smooth curves, γ_1 and γ_2, see Figure 1 (see next page) where one of the curves is highlighted.

   (a) Determine a parameterization of the planar curve defined by

$$x^2 + 4y^2 = 1$$

   (b) Extend the parameterization of the previous part to a parameterization of one of the curves γ_1, γ_2.

   (c) Determine the length of γ_1 and γ_2.

4. (a) Determine the Taylor Series Expansion, without error term, about $x = 0$ of $f(x) = \sqrt{1+x}$ up to and including order three.

   (b) Take the square of the polynomial that you found in the previous question. What strikes you?

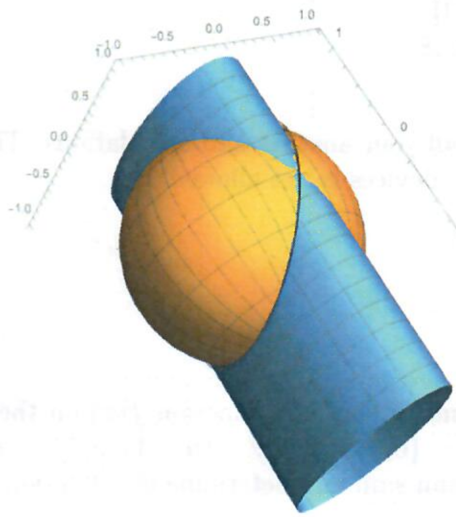

Figure 1: Intersection of a sphere and an elliptic cylinder.

5. Let $f(x, y)$ be given by

$$f(x, y) = xy^2 + x^2y - 6xy$$

   (a) Determine all critical points.
   (b) Determine the nature of the critical points, that is, (local) min/max, saddle point, ...
   (c) Determine the critical points of $f(x, y)$ on the curve defined by $xy = 1$.
   (d) Determine the nature of these critical points.
   (e) Assume that the equation $f(x, y) = 36$ defines, in a sufficiently small, neighborhood of $(x, y) = (1, 6)$, y as a function of x. Determine $y'(1)$.

Punten: **Ex 1**, a: 3, b: 4. **Ex 2**: a: 2, b: 4, **Ex 3**: a: 2, b: 3, c: 2. **Ex 4**: a: 3, b: 2. **Ex 5**: a: 2, b: 2, c: 3, d: 2, e: 2.