

Exam: Calculus II
Applied Mathematics & Applied Physics

Bachelors: AM & AP

Code 201800136/201800158

Date: 22 January 2020

Time: 08:45-11:45

Type of test closed book

Allowed aids nothing

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Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. Let S_n be given by

$$S_n = \sum_{k=1}^n \frac{(kn)^2 + k^4}{n^5}$$

- (a) Interpret S_n as a Riemann sum of a function $f(x)$ on the interval $[0, 1]$. Hint: take the partition $P_n = \{0, 1/n, 2/n, \dots, (n-1)/n, 1\}$ as the starting point for rewriting S_n as Riemann sum and determine the function $f(x)$.
- (b) Now calculate

$$\lim_{n \rightarrow \infty} S_n.$$

2. (a) Formulate the Mean Value Theorem for Triple Integrals.
(b) Let D be defined as

$$D = \{(x, y, z) \mid 0 \leq x^2 + y^2 \leq z, \quad 0 \leq z \leq 1\}$$

Calculate the volume of D .

- (c) Calculate the average value of $f(x, y, z) = z \sqrt{x^2 + y^2}$ on D .

3. Let the curve γ be given by

$$\gamma = \{(\cos^3(t), \sin^3(t)) \mid 0 \leq t \leq 2\pi\}.$$

(a) Prove that

$$\cos^4(t) \sin^2(t) + \cos^2(t) \sin^4(t) = \cos^2(t) \sin^2(t).$$

(b) Calculate the length of γ .

4. Let $f(x)$ be given by the powerseries:

$$f(x) = \sum_{k=0}^{\infty} \frac{x^{2k+4}}{k+1}$$

(a) Determine the radius of convergence of the powerseries.

(b) Prove that

$$f(x) = -x^2 \log(1 - x^2).$$

Hint: substitute $y = x^2$ in the powerseries and use the appropriate rules for powerseries to reduce the problem to a known powerseries.

5. Let $f(x, y)$ be given by

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \text{ for } (x, y) \neq (0, 0), \quad f(0, 0) = 0.$$

See Figure 1 for an impression of the graph of $f(x, y)$.

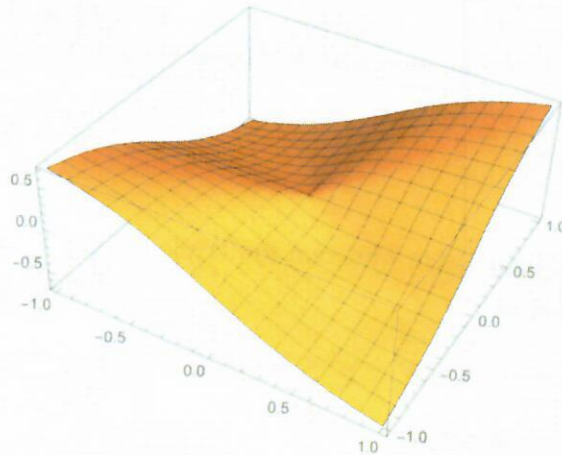


Figure 1: Graph of $f(x, y)$

- Give the definition of continuity of a function $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ in a point $(a, b) \in \mathbb{R}^2$.
- Investigate the continuity of $f(x, y)$ in $(0, 0)$.
- Find the extremal values, including their nature, of $f(x, y)$ on $[-1, 1] \times [-1, 1]$, that is the boundary of the square with corner points $(\pm 1, \pm 1)$.

Points:

Ex 1, a: 4, b: 3. **Ex 2**: a: 2, b: 2, c: 3. **Ex 3**: a: 2, b: 5. **Ex 4**: a: 2, b: 5. **Ex 5**: a: 1, b: 3, c: 4.

Grade:

$$1 + \frac{9P}{36}.$$