Exam: Calculus II
Applied Mathematics & Applied Physics

Bachelors: AM & AP

Code	201800136/201800158
Date:	31 January 2020
Time:	08:45-11:45
Type of test	closed book
Allowed aids	nothing

PTO/ZOZ

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Date : January 31st, 2020
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Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. Let S_n be given by

$$S_n = \sum_{k=1}^{n} \frac{4k}{n^3}\sqrt{4k^2+n^2}.$$

   (a) Interpret S_n as a Riemann sum of a function $f(x)$ on the interval $[0, 2]$.

   (b) Now calculate

$$\lim_{n\to\infty} S_n.$$

2. (a) Let the function $f(x)$ be given by $f(x) = x$, and let the domain $D \subset \mathbb{R}$ be given by

$$D = \{x \in \mathbb{R} \mid 0 \leq x \leq 1 \quad \text{or} \quad 2 \leq x \leq 3\}$$

   Determine the average value $\bar{f}$ of $f(x)$ on D.

   (b) Does there exist an $\bar{x} \in D$ sucht that $f(\bar{x}) = \bar{f}$.

   (c) Does your answer to 2b contradict the mean value theorem for integrals?

   (d) Let E be defined as

$$E = \{(x, y, z) \mid 1 \leq x^2 + y^2 + z^2 \leq 4\}$$

   Calculate the volume of E.

   (e) Calculate the average value of $f(x, y, z) = z\sqrt{x^2 + y^2 + z^2}$ on E.

3. Let the spatial curve γ be given by

$$\gamma = \left\{(t\cos(t), t\sin(t), \frac{2}{3}t\sqrt{2}\sqrt{t}) \mid 0 \leq t \leq 1\right\}.$$

   Calculate the length of γ.

4. Let $f(x) = \cos(2x)\sin(x^2)$.

   (a) Determine the Taylor Series appoximation of $f(x)$ about $x = 0$ of degree six.

   (b) Determine the radius of convergence of the powerseries.

5. Let $f(x, y, z)$ be given by

$$f(x, y, z) = xyz$$

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