**Calculus 2 for Applied Mathematics & Applied Physics**

| | |
|---|---|
| Code | 201800136/201800158 |
| Date: | 20 January 2021 |
| Time: | 09:00-12:00 |
| Type of test | closed book |
| Allowed aids | nothing |

Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. (4p) Calculate

$$\lim_{x\to\infty} (x - \sqrt{x^2 - 2x + 10}).$$

2. Let $S_n$ be given by

$$S_n = 2\sum_{k=1}^{n} \frac{\sqrt[3]{k}}{\sqrt[3]{n^4}}$$

   (a) (3p) Interpret $S_n$ as a Riemann sum of a function $f(x)$ on the interval $[0, 1]$. Hint: take the partition $P_n = \{0, 1/n, 2/n, \ldots, (n-1)/n, 1\}$ as the starting point for rewriting $S_n$ as Riemann sum and determine the function $f(x)$.

   (b) (3p) Now calculate

$$\lim_{n\to\infty} S_n.$$

3. The region $D \subset \mathbb{R}^2$ is the set enclosed by the lines

$$\ell_1 : x - y = 0 \quad \ell_2 : x - y = 1 \quad \ell_3 : x + y = 0 \quad \ell_4 : x + y = 1$$

   (a) Calculate

$$\iint_D x^2 - y^2 \,\mathrm{d}A$$

   in two different ways:

      i. (5p) Directly in Cartesian coordinates $(x, y)$.

      ii. (4p) Using the transformation $u = x - y$, $v = x + y$.

   (b) (3p) Calculate the average value of $f(x, y) = x^2 - y^2$ on $D$.

4. (4p) Let the spatial curve $\gamma$ be given by

$$\gamma = \left\{ (\frac{1}{3}t^3, t^2, 2t) \right\} \quad 0 \le t \le 2.$$

   Calculate the length of $\gamma$.

5. Let $f(x)$ be given by the power series:

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}(2x)^k$$

(a) (3p) Determine the radius of convergence of the power series.

(b) (4p) Determine the function $f(x)$.

6. Let $D$ be given by

$$\{(x, y) \mid x^2 + y^2 \leq 1\}$$

and $f(x, y)$ by

$$f(x, y) = x^4 - 2y^2$$

See Figure 1 for an impression of the graph of $f(x, y)$.

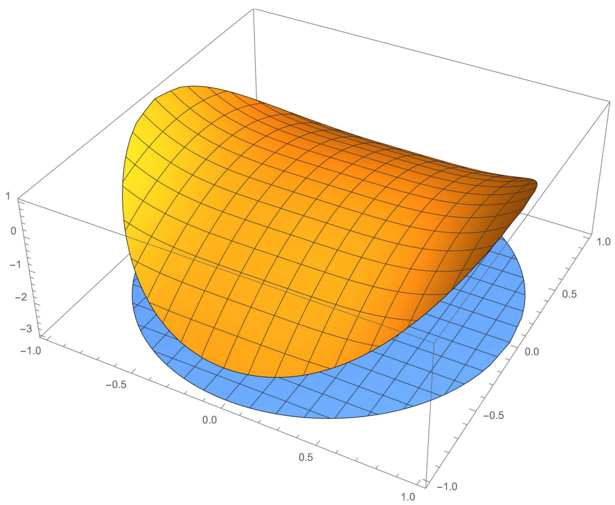


Figure 1: Graph of $f(x, y)$

(a) (2p) Determine the critical points of $f(x, y)$ in the interior of $D$.

(b) (5p) Show that $f(x, y)$ has a saddle point at the origin.

(c) (2p) Determine an equation of the tangent plane to the graph of $f(x, y)$ at the origin.

(d) (5p) Use the method of Lagrange Multipliers to find the extremal values, including their nature, of $f(x, y)$ on the boundary of $D$.

(e) (3p) What is the maximal directional derivative of $f(x, y)$ at the point $(\frac{1}{2}, \frac{1}{2})$?

Grade: $1 + \dfrac{9P}{50}$.