Calculus 2 for Applied Mathematics & Applied Physics

Code	201800136/201800158
Date:	26 January 2022
Time:	08:45-11:45
Type of test	closed book
Allowed aids	nothing

Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

There are six exam problems.

AM students: all answers should be formulated in English.

AP students: Dutch is allowed, English is preferred.

All students: at the end of the exam:

1. Put your student ID on the first page
2. scan your work with your smartphone
3. hand in your paper
4. convert your scan into a SINGLE DOCUMENT pdf
5. Upload the pdf on the Calculus 2 site of Canvas

Grade: $1 + \dfrac{9P}{66}$.

1. (9p) Calculate

$$\int \sin(x)^3 \mathrm{d}x.$$

2. Let S_n be given by

$$S_n = \sum_{k=1}^{n} \frac{1}{n+k}$$

(a) (5p) Interpret S_n as a Riemann sum of a function $f(x)$ on the interval $[0,1]$. Hint: take the partition $P_n = \{0, 1/n, 2/n, \ldots, (n-1)/n, 1\}$ as the starting point for rewriting S_n as Riemann sum and determine the function $f(x)$.

(b) (5p) Now calculate

$$\lim_{n\to\infty} S_n.$$

3. The region $D \subset \mathbb{R}^2$ is the set enclosed by the lines

$$\ell_1 : y = 1 \quad \ell_2 : y = 2 \quad \text{and the curves } \gamma_1 : y = \frac{1}{x} \quad \gamma_2 : y = \frac{-1}{x}$$

(a) (2p) Sketch D.

(b) Calculate the area of D in two different ways:

i. (2p) Using Cartesian coordinates (x, y).

ii. (3p) Using the coordinate transformation

$$u = y \quad v = xy$$

You will need to express x and y in terms of u and v.

(c) (2p) Define the function $f(x, y) = 4x^2 + y$. Prove that the function $f(x, y)$ together with the domain D satisfy the Mean Value Theorem for double integrals. Check all conditions.

(d) (3p) Calculate

$$\iint_D f(x, y)\, \mathrm{d}A$$

(e) (1p) Calculate the average value of $f(x, y)$ on D.

4. (10p) Let the spatial curve γ be given by

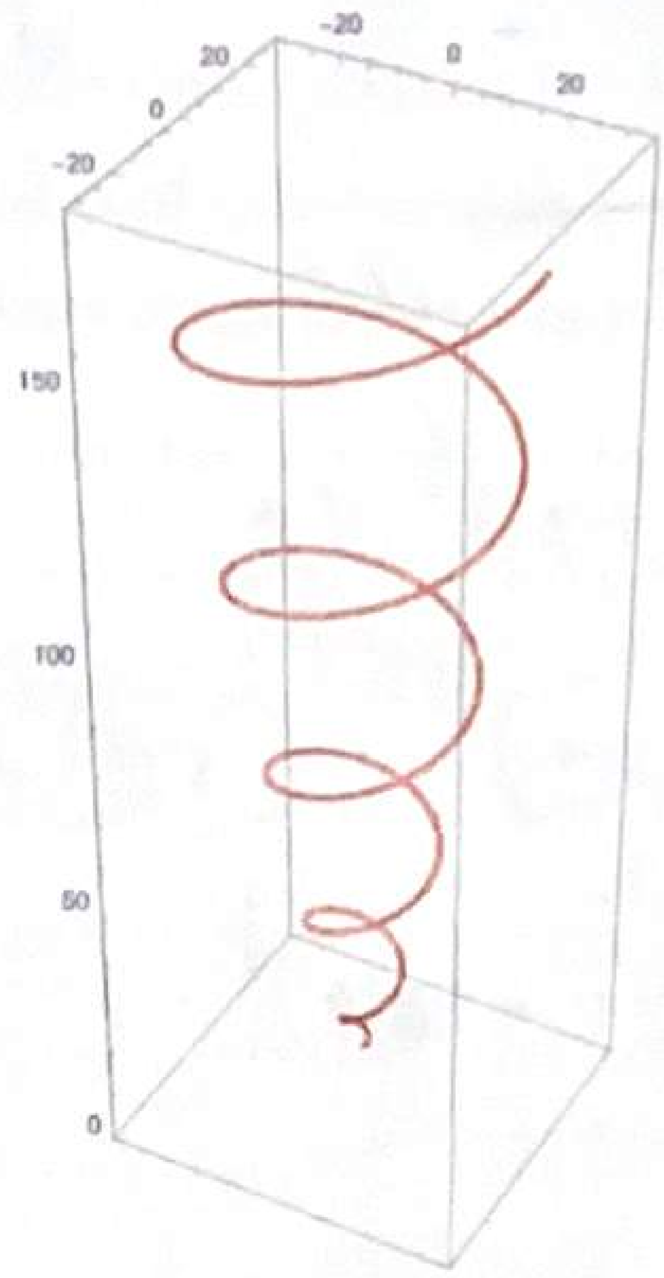

Figure 1: The curve γ

$$\gamma : \quad (t\cos(t), t\sin(t), \frac{2}{3}t\sqrt{2t}) \quad 0 \leq t \leq 10\pi.$$

See Figure 1. Calculate the length of γ.

5. Let $f(x) = \dfrac{1}{1+x^2}$.

(a) (4p) Determine the power series of $f(x)$.

(b) (3p) Determine the radius of convergence of the power series of $f(x)$.

(c) (4p) Determine the power series of $\arctan(x)$.

6. Let $f(x, y)$ be given by

$$f(x, y) = x^3 y + x y^3.$$

See Figure 2 for an impression of the graph of $f(x, y)$.

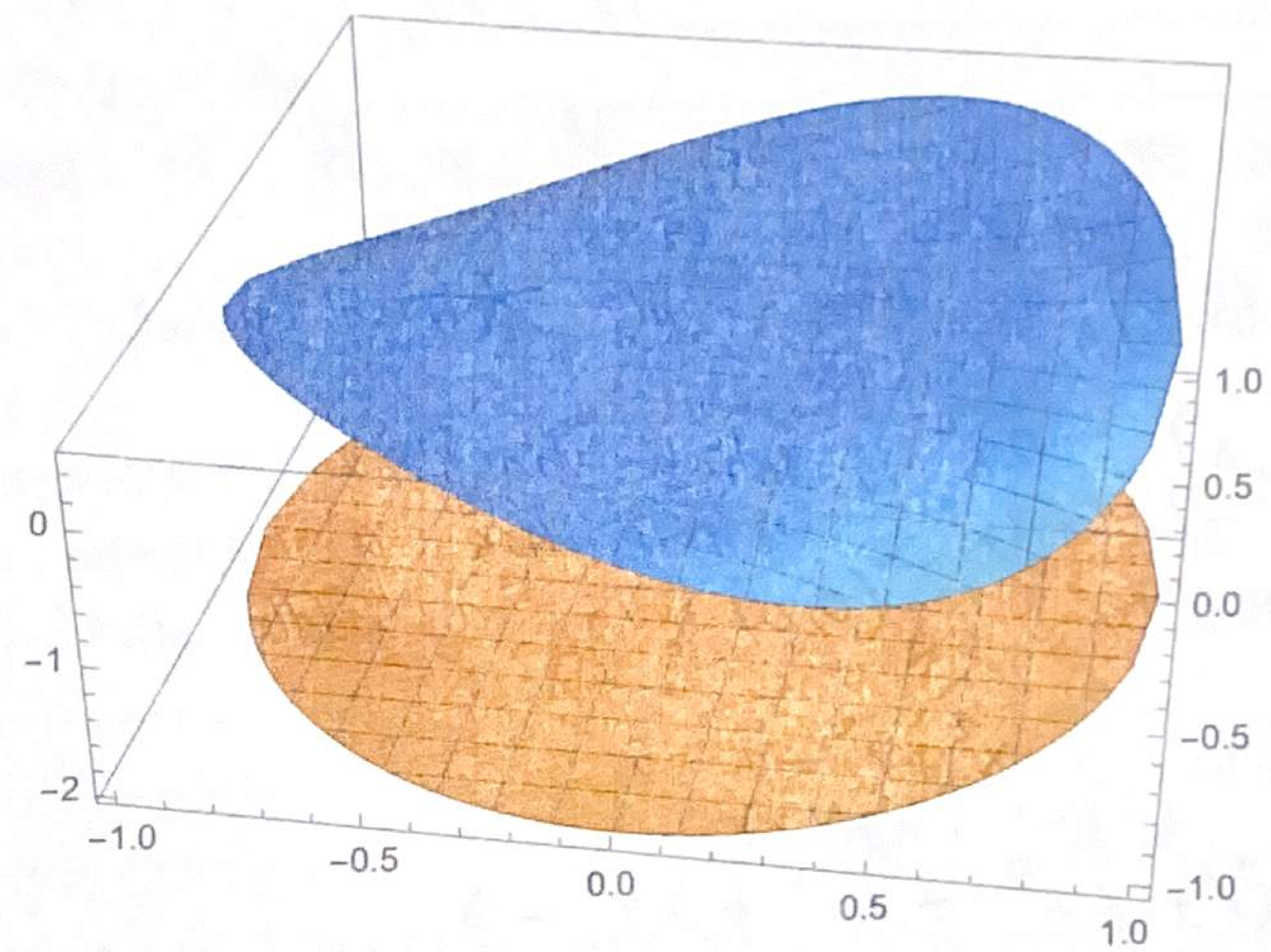

Figure 2: Graph of $f(x, y)$

(a) (2p) Calculate grad$f(x, y)$.

(b) (2p) Determine all critical points.

(c) (2p) Determine the nature of the critical points, that is, (local) min/max, saddle point. Hint: you may want to consider $f(x, x)$ and $f(x, -x)$.

(d) (4p) Use Lagrange multipliers to determine the critical points of $f(x, y)$ on the curve defined by $x^2 + y^2 = 1$.

(e) (3p) Determine the nature of these critical points.