

Kenmerk : TW2013/TW/DWMP/034/gp

Course : **Mathematics B2: Newton**

Datum : 29 november 2013

Tijd : 15.45 - 16.45

**Motiveer alle antwoorden en berekeningen.
Gebruik van elektronische hulpmiddelen is niet toegestaan.**

1. De functie $f : \mathbb{R} \rightarrow \mathbb{R}$ is gegeven door:

$$f(x) = \begin{cases} \frac{x}{x^2 + 1} & \text{als } x < 0; \\ x + 2\sqrt{x} & \text{als } x \geq 0. \end{cases}$$

[3 pt] (a) Toon aan dat f continu is in 0.

[2 pt] (b) Bepaal

$$\lim_{x \rightarrow \infty} f(x) \text{ en } \lim_{x \rightarrow -\infty} f(x).$$

[4 pt] (c) Bepaal de absolute extrema van f op het interval $[-2, 2]$. Motiveer duidelijk alle stappen.

[3 pt] 2. Bereken de volgende limiet met behulp van de regel van L'Hôpital.

$$\lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{1}{x}}.$$

3. De functie $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ wordt gegeven door:

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2} & \text{als } (x, y) \neq (0, 0) \\ 0 & \text{als } (x, y) = (0, 0). \end{cases}$$

[3 pt] (a) Onderzoek of f continu is in $(0, 0)$.

[3 pt] (b) Bepaal de vergelijking van het raakvlak aan de grafiek van f in het punt $(1, 1, \frac{1}{2})$.

Totaal 18 punten.

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**Motivate all your answers and calculations.
Use of electronic devices is not allowed.**

1. The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by:

$$f(x) = \begin{cases} \frac{x}{x^2 + 1} & \text{if } x < 0; \\ x + 2\sqrt{x} & \text{if } x \geq 0. \end{cases}$$

[3 pt] (a) Show that f is continuous at 0.

[2 pt] (b) Determine

$$\lim_{x \rightarrow \infty} f(x) \text{ and } \lim_{x \rightarrow -\infty} f(x).$$

[4 pt] (c) Find the absolute extrema of f on the interval $[-2, 2]$. Give a clear motivation of all steps.

[3 pt] 2. Calculate the limit below by using L'Hôpital's rule.

$$\lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{1}{x}}.$$

3. The function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is given by:

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

[3 pt] (a) Investigate whether f is continuous at $(0, 0)$.

[3 pt] (b) Find the equation for the tangent plane to the graph of f at the point $(1, 1, \frac{1}{2})$.

In total 18 points.