Kenmerk : TW2014/TW/DWMP/010/gp

Course : **Mathematics B2: Newton**

Datum : 27 januari 2014
Tijd : 8.45 - 10.45

Motiveer alle antwoorden en berekeningen.
Gebruik van elektronische hulpmiddelen is niet toegestaan.

Eerste deel (18 punten)

[3 pt] 1. Bereken de volgende limiet:

$$\lim_{x\to\infty} \frac{\sqrt{2x^2-6x+2}}{x+3}$$

2. De functie $f : [-1,1] \to \mathbb{R}$ is gegeven door:

$$f(x) = \begin{cases} x\ln(x^2) & \text{als } x \neq 0 \\ 0 & \text{als } x = 0. \end{cases}$$

[1 pt] (a) Geef de definitie van "f is continu in 0".

[2 pt] (b) Toon aan dat f continu is in 0.

[4 pt] (c) Bepaal de absolute extrema van f op het interval [-1,1]. Motiveer duidelijk alle stappen.

3. De functie $f : \mathbb{R}^2 \to \mathbb{R}$ wordt gegeven door:

$$f(x,y) = \begin{cases} \dfrac{2x^2+y^5}{x^2+y^4} & \text{als } (x,y) \neq (0,0) \\ 0 & \text{als } (x,y) = (0,0). \end{cases}$$

[3 pt] (a) Onderzoek of f continu is in $(0,0)$.

[3 pt] (b) Bepaal de partiële afgeleiden $\dfrac{\partial f}{\partial x}(x,y)$ en $\dfrac{\partial f}{\partial y}(x,y)$ voor $(x,y) \neq (0,0)$.

[2 pt] (c) Bepaal het raakvlak aan de grafiek van f in het punt $(0,1,1)$.

Tweede deel (18 punten)

[3 pt] 4. Bepaal $\frac{dy}{dx}$ als $y = x \int_1^x \frac{t}{1+t^4} dt$.

[4 pt] 5. (a) Bepaal $\int_0^\infty \frac{x}{(1+x^2)^2} dx$.

[4 pt] (b) Bepaal $\int x \ln^2(x) dx$. Hint: pas twee keer partiële integratie toe.

[3 pt] 6. Vind het convergentie-interval van

$$\sum_{n=0}^{\infty} (x^2 - \frac{1}{2})^n$$

en bereken de som voor alle x uit dat interval.

[4 pt] 7. Bepaal de Taylorreeks van $f(x) = \cos(2x)$ rondom $x = 0$.

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Motivate all your answers and calculations.
Use of electronic devices is not allowed.

First part (18 points)

[3 pt] 1. Find the limit:

$$\lim_{x\to\infty} \frac{\sqrt{2x^2-6x+2}}{x+3}$$

2. The function $f : [-1,1] \to \mathbb{R}$ is given by:

$$f(x) = \begin{cases} x\ln(x^2) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

[1 pt] (a) Give the definition of "f is continuous at 0".

[2 pt] (b) Show that f is continuous at 0.

[4 pt] (c) Find the absolute extrema of f on the interval [-1,1]. Give a clear motivation of each step.

3. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by:

$$f(x,y) = \begin{cases} \dfrac{2x^2+y^5}{x^2+y^4} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0). \end{cases}$$

[3 pt] (a) Investigate whether f is continuous at $(0,0)$.

[3 pt] (b) Find the partial derivatives $\dfrac{\partial f}{\partial x}(x,y)$ and $\dfrac{\partial f}{\partial y}(x,y)$ for $(x,y) \neq (0,0)$.

[2 pt] (c) Find the tangent plane to the graph of f at the point $(0,1,1)$.

Second part (18 points)

[3 pt] 4. Find $\frac{dy}{dx}$ for $y = x \int_1^x \frac{t}{1+t^4} dt$.

[4 pt] 5. (a) Find $\int_0^\infty \frac{x}{(1+x^2)^2} dx$.

[4 pt] (b) Find $\int x \ln^2(x) dx$. Hint: apply partial integration twice.

[3 pt] 6. Find the interval of convergence of

$$\sum_{n=0}^{\infty} (x^2 - \frac{1}{2})^n$$

and find the sum for all x in this interval.

[4 pt] 7. Find the Taylor series of $f(x) = \cos(2x)$ at $x = 0$.