Exam Vector Calculus for Applied Physics/Applied Mathematics Bachelor Module 4

Codes 201300164, 201400535
May 22, 2017, 8.45-11.45

- **All answers must be motivated and clearly formulated.**
- **The use of a calculator is not allowed.**

1. Given the planes: $S_1 : x + 2y + z = 2$ and $S_2 : 3x - y - 2z = 4$.

   a. Calculate the parametrization $\mathbf{r}_1(t)$ of the intersection line of the planes S_1 and S_2.

   b. Calculate the cosine of the angle between the planes S_1 and S_2.

   Given the curves: $\mathbf{r}_2(t) = (t, t^2, t^3)$ and $\mathbf{r}_3(t) = (\sin(\pi t/2), (2-t)^2, 2t^2 - 1)$.

   c. Calculate a vector that is normal to the tangent vectors of the curves $\mathbf{r}_2$ and $\mathbf{r}_3$ at the point of intersection $(1, 1, 1)$.

2. Given the functions

$$z(x, y) = \ln\left(\frac{1}{1 + x + 2y}\right), \quad x(s, t) = e^{st}, \quad y(t) = \cos t.$$

   a. Calculate $\frac{\partial z}{\partial s}$.

   b. Calculate $\frac{\partial^2 z}{\partial t \partial s}$.

   Given the function $f(x, y) = \sqrt{1 + \exp(x - 2y - 3)}$, with exp the exponential function.

   c. Calculate the Taylorseries of $f(x, y)$ around the point $(x, y) = (1, -1)$ up to and including the linear terms.

   d. Calculate the directional derivative of $f(x, y)$ at $(x, y) = (1, -1)$ in the direction of the vector $\mathbf{u} = (\sqrt{2}, \sqrt{2})$.

3. Calculate the integral

$$\iint_R \sqrt{2x(y - 2x)}\, dA,$$

   where R is the parallelogram in the xy-plane with vertices $(0, 0)$, $(0, 1)$, $(2, 4)$, and $(2, 5)$.

   a. Calculate the integral using the transformation $T : x = 2u, y = 4u + v$.

b. Calculate the integral directly in the $x-y$ plane, thus without using a coordinate transformation.

4. Calculate the integral

$$\iint_S \sqrt{x^2+y^2}dS,$$

with S the surface given by the parameterization

$$\mathbf{r}(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + \frac{2}{3}v^{\frac{3}{2}}\mathbf{k} \qquad 0 \le u \le 1,\ 0 \le v \le 1.$$

5. Compute the integral

$$\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}, = \int_C F\cdot dr$$

with the vector field $\mathbf{F}$ given by

$$\iint_S F \cdot \hat{N}\, dS = \iiint_D \operatorname{div} F\, dP$$

$$\mathbf{F}(x,y,z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + zx\mathbf{k}.$$

The surface S is that part of the sphere $x^2+y^2+z^2=5$ the lies above the plane the plane $z=1$. The surface S has a normal vector with a positive component in the z-direction of the Cartesian coordinate system.

6. Investigate if the following series converge or diverge

a. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^{\frac{3}{2}}}$ b. $\displaystyle\sum_{n=1}^{\infty}\frac{n+9^n}{n+10^n}$ c. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2 2^n}{n!}$

Grading

1: 6	2: 6	3: 5	4: 6	5: 7	6: 6
1a: 2	2a: 1	3a: 3			6a: 2
1b: 2	2b: 2	3b: 2			6b: 2
1c: 2	2c: 2				6c: 2
	2d: 1				

total 36+4=40 points