Exam Vector Calculus for Applied Physics/Applied Mathematics Bachelor Module 4

Codes 201300164, 201400535
February 23, 2018, 8.45-11.45

- **All answers must be motivated and clearly formulated.**
- **The use of a calculator is not allowed.**

1. Given the plane $S : 2x + 4y - 3z = 30$ and the point $P = (1, 2, 3)$.

   a. Give the parametrization of the line L passing through the point P and normal to the plane S.

   b. What is the shortest distance between the point P and the plane S?

   c. Given the curve $C : \mathbf{r}_C(t) = \sin(\frac{\pi}{2}t)\mathbf{i} + (2 + \cos(\frac{\pi}{2}t))\mathbf{j} + 3t^3\mathbf{k}$.

   What is cosine of the angle between the line L and the tangent vector at the curve C at their intersection point $\mathbf{r}_C(1) = (1, 2, 3)$?

2. Given the function

$$z(x, y) = \sin(x \cos y)$$

   with

$$x(s, t) = t\sqrt{s} \quad \text{and} \quad y(s, t) = \sin(\sqrt{t})$$

   a. Calculate $\frac{\partial z}{\partial t}$. Note, you are **not allowed** to eliminate the x and y variables.

   b. Calculate $\frac{\partial^2 z}{\partial s \partial t}$.

3. Investigate if the following series converge or diverge

$$a. \sum_{n=1}^{\infty} \frac{n^3 e^n}{n!} \qquad b. \sum_{n=1}^{\infty} \frac{\sqrt{n}\sin^2 n}{(n+1)^2 + \log n} \qquad c. \sum_{n=3}^{\infty} \left(\frac{3 \log n}{\log(n^2 - 4)} \right)^n$$

4. Calculate the integral $\iint_D x dA$ on the domain

$$D := \{(x, y) \in \mathbb{R}^2 \ : \ x = r^2 \cos\theta, \ y = r\sin^2\theta, \ 0 \leq \theta \leq \frac{\pi}{2}, 1 \leq r \leq 2\}.$$

5. Given the domain

$$D := \{(x, y, z) \in \mathbb{R}^3 \ : \ y \leq 1 - x^2 - z^2, \ 0 \leq y \leq 1\}.$$

At the surface S of the domain D the unit outward normal vector is denoted as $\hat{\mathbf{N}}$.

Given the vector field $\mathbf{F}(x, y, z) = (x^2 y + \log(1 + z^2))\mathbf{i} + y^2\mathbf{j} + z\mathbf{k}$.

a. Is $\mathbf{F}$ a solenoidal field? Motivate your answer.

b. Calculate $\iint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS$.

6. Given the surface

$$S := \{(x, y, z) \in \mathbb{R}^3 \ : \ x^2 + y^2 + z^2 = 4, \ y \geq 0, \ z \geq 0\}.$$

The unit normal vector $\hat{\mathbf{N}}$ at S has positive or zero z-component. Given the vector field

$$\mathbf{F}(x, y, z) = x^2 y\mathbf{i} + zy\mathbf{k}.$$

a. Calculate **curl F**.

b. Is $\mathbf{F}$ a conservative field? Motivate your answer.

c. Calculate $\iint_S \textbf{curl } \mathbf{F} \cdot \hat{\mathbf{N}} dS$ using Stokes' theorem.

d. Calculate $\iint_S \textbf{curl } \mathbf{F} \cdot \hat{\mathbf{N}} dS$ without using Stokes' theorem.

Grading

1: 5	2: 6	3: 5	4: 6	5: 6	6: 8
1a: 1	2a: 1	3a: 2		5a: 1	6a: 1
1b: 2	2b: 3	3b: 3		5b: 5	6b: 1
1c: 2	2c: 2				6c: 3
					6d: 3

total 36+4=40 points