Exam Vector Calculus for Applied Mathematics Bachelor Module 4

Code 201400535

June 25, 2018, 8.45-10.45

- **All answers must be motivated and clearly formulated.**
- **The use of a calculator is not allowed.**

1. Given the domain

$$D = \{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^4 \leq 1, \ x \geq 0, \ 0 \leq y \leq \sqrt{x}\}.$$

Calculate

$$\iint_D y \sin(x^2 + y^4) dA.$$

Use the transformation

$$\begin{aligned} x &= r\cos\theta, \\ y &= \sqrt{r\sin\theta} \end{aligned}$$

2. Given the surface S with parametrization

$$\mathbf{r}(u, v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + \frac{2}{3}v^{\frac{3}{2}}\mathbf{k}, \qquad \text{with } 0 \leq u \leq 1, 0 \leq v \leq 1.$$

Calculate

$$\iint_S \sqrt{x^2 + y^2} dS.$$

3. Given the vector field

$$\mathbf{F}(x, y, z) = x^2 y\mathbf{i} + y\mathbf{j} + e^{xy}\mathbf{k}.$$

Given the surface

$$S = \{(x, y, z) \in \mathbb{R}^3 \ : \ x^2 + y^2 + z^2 = 4, \ y \geq 0, \ z \geq 0\}.$$

The surface S has a unit normal vector $\hat{\mathbf{N}}$ with a negative or zero z-component.

Use Stokes' theorem to calculate

$$\iint_S \mathbf{curl}\,\mathbf{F} \cdot \hat{\mathbf{N}} dS.$$

4. Given the domain

$$D = \{(x, y, z) \in \mathbb{R}^3 \ : \ x^2 + y^2 + z^2 \leq 8, x \geq 0, z \geq 2\}.$$

At the surface S of the domain D the unit outward normal vector is $\hat{\mathbf{N}}$. Given the vector field

$$\mathbf{F}(x, y, z) = y^2\mathbf{i} + zy\mathbf{j} + xy\mathbf{k}.$$

Calculate

$$\iint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS.$$

Grading

1: 6	2: 6	3: 6	4: 6

total 24+3=27 points