Exam Vector Calculus

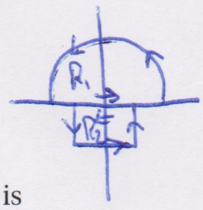
Module 3 Applied Physics and Applied Mathematics Bachelor

Codes 201700164-201900257

February 28, 2020, 8.45-10.45

- **All answers must be motivated and clearly formulated.**
- **The use of a calculator is not allowed.**

1. The domain $R = R_1 \cup R_2$ with positively oriented boundary C consists of the union of two subdomains:



$$R_1 := \{(x,y) \in \mathbb{R}^2 \;:\; 0 \le x^2 + y^2 \le 4,\; y \ge 0\},$$
$$R_2 := \{(x,y) \in \mathbb{R}^2 \;:\; -1 \le x \le 1,\; -1 \le y \le 0\}.$$

The domains R_1 and R_2 both have a positively oriented boundary, which is denoted, respectively, as C_1 and C_2.

   a. Motivate why you can use Greens' theorem on the domain R if you can apply Green's theorem to R_1 and R_2 separately.

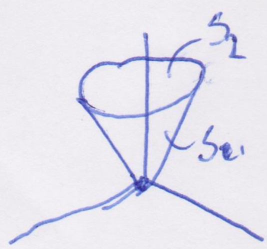
   b. Given the vector field $\mathbf{F}(x,y) := x^3 y^2 \mathbf{i} + x y^2 \mathbf{j}$.

   Use Green's theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

   c. Given the vector field $\mathbf{F}(x,y) := \pi \frac{\partial \phi}{\partial x} \mathbf{i} + \pi \frac{\partial \phi}{\partial y} \mathbf{j}$, with ϕ an arbitrary twice continuously differentiable function on the domain R.

   Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

2. Given a cone in $\mathbb{R}^3$ with boundary surface $S = S_1 \cup S_2$, where



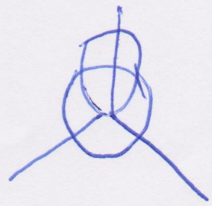
$$S_1 := \{(x,y,z) \in \mathbb{R}^3 \;:\; z = 2\sqrt{x^2 + y^2},\; 0 \le z \le 2\}$$
$$S_2 := \{(x,y,z) \in \mathbb{R}^3 \;:\; x^2 + y^2 \le 1,\; z = 2\},$$

and unit outward normal vector $\hat{\mathbf{N}}$.

Given the vector field
$$\mathbf{F}(x,y,z) := xz\mathbf{i} + yz^3\mathbf{j}.$$

   a. Calculate $\oiint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS$ using Gauss' theorem.

   b. Calculate $\oiint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS$ directly using the surface parametrization.



3. Given the contour C with

$$C := \{(x, y, z) \in \mathbb{R}^3 : \; x^2 + y^2 = 4, \; x + z = 4\}.$$

The orientation of C is counter clockwise when viewed from large values of z along the z-axis.

Given the vector field

$$\mathbf{F}(x, y, z) := (z^2 + \sin(x^2))\mathbf{i} + (2xy + z)\mathbf{j} + (xz + 2yz)\mathbf{k}.$$

Calculate $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

Grading

1: 10	2: 10	3: 7
1a: 2	2a: 5	3: 7
1b: 5	2b: 5	
1c: 3		

total 27+3=30 points