Exam Vector Calculus
Module 3 Applied Physics and Applied Mathematics Bachelor

Codes 202001229-202001232

February 19, 2021, 9.00-11.00

The use of a book is not allowed

All answers must be justified and clearly formulated.

1. Given a triangular domain D with vertices $(0,0), (1,0)$ and $(0,1)$ in a Cartesian coordinate system. The boundary of the domain D is denoted as C and has a positive orientation.

   Given the vector field

   $$\mathbf{F}(x,y) = F_1(x,y)\mathbf{i} + F_2(x,y)\mathbf{j} := (1-x)^2 y\mathbf{i} + \frac{1}{2}(1-x)y^2\mathbf{j}.$$

   a. Sketch the domain D. Indicate the proper orientation of C.

   b. Use Green's theorem to compute $\oint_C F_1(x,y)dx + F_2(x,y)dy$.

   c. Calculate $\oint_C F_1(x,y)dx + F_2(x,y)dy$ directly without using Green's theorem.

   d. Compute $\oint_C \mathbf{F}\cdot\hat{\mathbf{N}}ds$, with $\hat{\mathbf{N}}$ the exterior unit normal vector at C.

   e. Is the vector field $\mathbf{F}$ solenoidal? Please justify your answer.

2. The domain $R \subset \mathbb{R}^3$ has the boundary surface S that consists of the parts

   $$\begin{aligned} S_1 &:= \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2 = 4,\ 0 \le z \le 2+y\},\\ S_2 &:= \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2 \le 4,\ z = 0\},\\ S_3 &:= \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2 \le 4,\ z = 2+y\}. \end{aligned}$$

   At the boundary surface S the external unit normal vector is given by $\hat{\mathbf{N}}$. Given the vector field

   $$\mathbf{F}(x,y,z) := x^2 y\mathbf{i} + xy^2\mathbf{j} + e^{x^2}\mathbf{k}.$$

   a. Sketch the surface S.

   b. Calculate $\iint_S \mathbf{F}\cdot\hat{\mathbf{N}}dS$.

3. Given the surface S, defined as

   $$S := \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2+z^2 = 4,\ z \ge \sqrt{2}\}.$$

The boundary of the surface S is denoted as C. The unit normal vector $\hat{\mathbf{N}}$ at S has a positive z-component.

Given the vector field

$$\mathbf{F}(x, y, z) := x^2 y\mathbf{i} + y\mathbf{j} + yz\mathbf{k}.$$

a. Sketch the surface S. Indicate the orientation of the boundary of S.
b. Give the parametrization of the surface S.
c. Show that $\text{curl}\,\mathbf{F} = z\mathbf{i} - x^2\mathbf{k}$.
d. Calculate $\iint_S \textbf{curl}\,\mathbf{F} \cdot \hat{\mathbf{N}} dS$ using the parametrization of the surface S.
e. Calculate $\iint_S \textbf{curl}\,\mathbf{F} \cdot \hat{\mathbf{N}} dS$ using Stokes' theorem.

- Please do the following after you completed your exam:

1. After getting permission of the invigilators, please use your mobile phone to take pictures of your answer sheets.
2. Hand-in your paper answer sheets to the invigilators.
3. Upload **before February 19, 13.30 hours** the pictures of your answers, your student card, and if applicable your card for extra time, **as one pdf file** in the Assignment Section of the Mod 03 TN/AM Joint Parts Canvas site:

   Mod 03 TN/AM Joint Parts (2020-2A)\Assignments\Vector Calculus Exam-19 February- PDF UPLOAD

Other formats than pdf will not be accepted.

Please make sure your scan and answers are readable. What we cannot read, we cannot grade.

Grading

1: 12	2: 7	3: 17
1a: 1	2a: 1	3a: 1
1b: 3	2b: 6	3b: 3
1c: 4		3c: 1
1d: 3		3d: 6
1e: 1		3e: 6

total 36+4=40 points