E&M
UNIVERSITY OF TWENTE.

Partial test II E&M
2 April 2019

Offerhaus, Marpaung, Dhallé
2017-201400535-2B
Fields and electromagnetism

Instructions You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in. You may use a hand-written formula sheet containing maximum 10 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (15 points). Consider a short solenoidal coil that carries a current I_f (Figure 1).

1.a. Copy figure 1a) and sketch the field lines of the magnetic induction **B**. Clearly indicate the shape and the direction of the field lines both inside the coil as well as outside..

Now a cylindrical paramagnetic rod ($\chi_m > 0$) is partially placed inside (and coaxial with) the coil.

1.b. Copy figure 1b) and sketch the bound magnetisation currents $\mathbf{K}_b$ on the surface of the rod. Again, indicate their shape and direction.

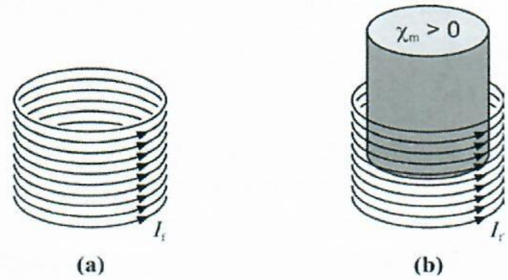

Figure 1a) short cylindrical current-carrying solenoid and **b)** the same coil, this time partially filled with a paramagnetic cylinder.

1.c. Include in your sketch of problem 1b) the Lorentz force $\mathbf{F}_L$ that acts on the magnetization currents just above the coil. Clearly indicate the direction of the force. Is the rod pulled further into the coil, or is it expelled out of the coil?

Problem 2 (15 points). Below you find five statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

2.a. Two parallel straight wires that carry a current in the same direction repel each other.

2.b. In *static* situations, the curl of the magnetic vector potential is always zero.

2.c. When one places a slab of a paramagnetic material (μ_r>1) in a magnetic field that is not perpendicular to its surface, the magnetic field lines inside the slab bend towards the normal direction.

2.d. The self-inductance of a coil is proportional to the current that flows in the coil.

2.e. When one doubles the current flowing in a circuit, the magnetic energy associated with this current also doubles..

Problem 3 (10 points). A homogeneous magnetic field with flux density $\mathbf{B} = B_0\hat{\mathbf{y}}$ points in the positive y-direction (Figure 2).

Each of the six vectors $\mathbf{v}_1 - \mathbf{v}_6$ in figure 2 represents the velocity of a point charge q. All six vectors have the same magnitude v_0 and the 'enveloping' figure represents a cube. For each of these six velocity directions, write down an expression for the force **F** experienced by the charge (magnitude and direction). This expression should only contain magnitudes and Cartesian coordinates, no vector products.

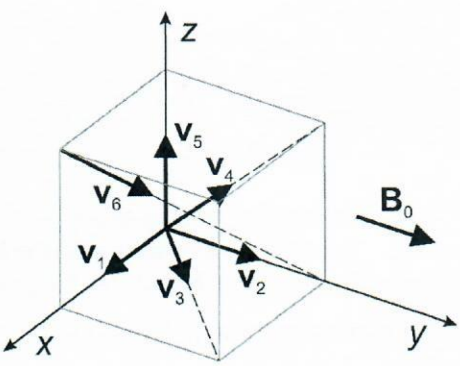

Figure 2: Different velocities $\mathbf{v}_i$ of a charge q moving in a uniform field $\mathbf{B}_0$.

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Problem 4 (20 points). Two long wires lie in the *xy*-plane, parallel to the *x*-axis and at a distance $2R$ apart (Figure 3). The bottom one (at $y = -R$) carries a current I from $x = -\infty$ to $x = 0$. At $x > 0$, a half-circular wire loop with radius R turns the current around and sends it back along the top wire (at $y = +R$) from $x = 0$ to $x = -\infty$.

4.a. What is the contribution of the bottom wire to the total **B**-field in the origin $(x,y,z) = (0,0,0)$ (give magnitude and direction)?

4.b. What is the contribution of the top wire?

4.c. What is the contribution of the half-circle?

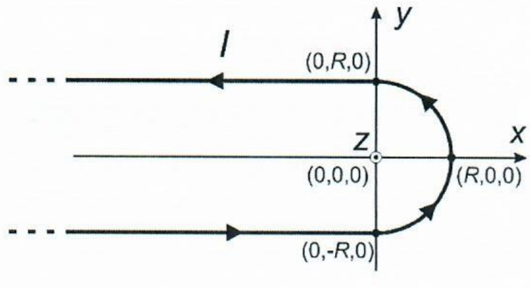

Figure 3: Current circuit around the origin (Problem 4).

Problem 5 (25 points). A long solid copper cylinder (with magnetic permeability $\mu_r \approx 1$ and radius R_1) carries a current I_0 in its axial direction. The current is distributed uniformly over its cross-section. The copper cylinder is tightly surrounded by an iron sleeve (a hollow cylinder with $\mu_r = 1000$, inner radius R_1 and outer radius R_2). The cylinder is much longer than its diameter so that effects of the finite length may be neglected.

5.a. Work out the magnitude and direction of the magnetic flux density **B** inside the copper ($s < R_1$), inside the iron ($R_1 < s < R_2$) and in the empty space outside the iron ($s > R_2$).

5.b. Work out an expression for the magnetic vector potential **A** in these three regions (magnitude and direction).

5.c. Work out all bound currents $\mathbf{K}_b$ and $\mathbf{J}_b$ (magnitude and direction).

Problem 6 (15 points). A long cylindrical air-filled coil with n turns per unit length and radius a caries a current $I_{coil} = I_0$. One single 'pick-up' loop is wound around the coil and closed with a resistor R (Figure 4).

At the time $t = 0$, one starts to ramp down the current I_{coil} at a constant rate. After a time t_0, the current I_{coil} is zero:

$$I_{coil}(t) = I_0 \qquad (t \le 0)$$
$$I_{coil}(t) = I_0\left(1 - \frac{t}{t_0}\right) \qquad (0 \le t \le t_0)$$
$$I_{coil}(t) = 0 \qquad (t_0 \le t)$$

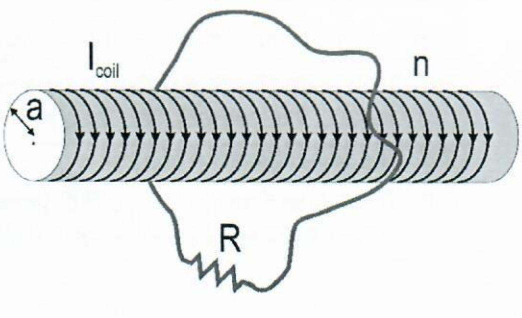

Figure 4: current-carrying coil with a resistive pick-up loop around it (Problem 6).

6.a. During the ramping-down, what is the magnitude of the current I_{loop} that is induced in the pick-up loop? In the view of Figure 4, does I_{loop} flow from the front to the back of the resistor or from its back to its front?

6.b. How much heat U [J] is released in the resistor R during the ramp-down?

6.c. The current is ramped up again to $I_{coil} = I_0$, an iron core ($\mu_r = 1000$) is inserted tightly into the coil (whilst keeping I_{coil} constant) and – after a pause to let all transient currents in the pick-up loop die out – the ramp-down experiment is repeated. How much heat is released in R this time?

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}};\quad d\tau = dx\,dy\,dz$

$$\textit{Gradient}: \quad \nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\textit{Divergence}: \quad \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\textit{Curl}: \quad \nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\textit{Laplacian}: \quad \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}};\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

$$\textit{Gradient}: \quad \nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\textit{Divergence}: \quad \nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\textit{Curl}: \quad \nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

$$\textit{Laplacian}: \quad \nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}};\quad d\tau = s\,ds\,d\phi\,dz$

$$\textit{Gradient}: \quad \nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\textit{Divergence}: \quad \nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\textit{Curl}: \quad \nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

$$\textit{Laplacian}: \quad \nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

FUNDAMENTAL CONSTANTS

ϵ_0	$=$	$8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$	(permittivity of free space)
μ_0	$=$	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$	(permeability of free space)
c	$=$	$3.00 \times 10^{8}\,\mathrm{m}/s$	(speed of light)
e	$=$	$1.60 \times 10^{-19}\,\mathrm{C}$	(charge of the electron)
m	$=$	$9.11 \times 10^{-31}\,\mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\left\{\begin{array}{lcl} x &=& r\sin\theta\cos\phi \\ y &=& r\sin\theta\sin\phi \\ z &=& r\cos\theta \end{array}\right. \qquad \left\{\begin{array}{lcl} \hat{\mathbf{x}} &=& \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} &=& \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} &=& \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{array}\right.$$

$$\left\{\begin{array}{lcl} r &=& \sqrt{x^2+y^2+z^2} \\ \theta &=& \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi &=& \tan^{-1}(y/x) \end{array}\right. \qquad \left\{\begin{array}{lcl} \hat{\mathbf{r}} &=& \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} &=& \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} &=& -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{array}\right.$$

Cylindrical

$$\left\{\begin{array}{lcl} x &=& s\cos\phi \\ y &=& s\sin\phi \\ z &=& z \end{array}\right. \qquad \left\{\begin{array}{lcl} \hat{\mathbf{x}} &=& \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} &=& \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} &=& \hat{\mathbf{z}} \end{array}\right.$$

$$\left\{\begin{array}{lcl} s &=& \sqrt{x^2+y^2} \\ \phi &=& \tan^{-1}(y/x) \\ z &=& z \end{array}\right. \qquad \left\{\begin{array}{lcl} \hat{\mathbf{s}} &=& \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} &=& -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} &=& \hat{\mathbf{z}} \end{array}\right.$$

Standaardintegralen .

$I=\int x^m (a^2+x^2)^n\,dx$ Noem $Y=\sqrt{a^2+x^2}$; $Y^2=a^2+x^2$;

m	*n*	*I*	*m*	*n*	*I*
-2	-1/2	$-Y/(a^2x)$	1	-3/2	$-1/Y$
-2	-1	$-a^{-2}(\frac{1}{x}+\frac{1}{a}\arctan\frac{x}{a})$	1	-1	$\ln\lvert Y\rvert$
-1	-3/2	$a^{-2}(\frac{1}{Y}-\frac{1}{a}\ln\left\lvert\frac{a+Y}{x}\right\rvert)$	1	-1/2	Y
-1	-1/2	$-(1/a)\ln\lvert(a+Y)/x\rvert$	1	1/2	$\frac{1}{3}Y^3$
-1	-1	$a^{-2}\ln\lvert x/Y\rvert$	1	3/2	$\frac{1}{5}Y^5$
0	-3/2	$x/(a^2Y)$	2	-3/2	$\ln\lvert x+Y\rvert-x/Y$
0	-1	$a^{-1}\arctan(x/a)$	2	-1	$x-a\arctan(x/a)$
0	-1/2	$\ln\lvert x+Y\rvert$	2	-1/2	$\frac{1}{2}xY-\frac{1}{2}a^2\ln\lvert x+Y\rvert$
0	1/2	$\frac{1}{2}xY+\frac{1}{2}a^2\ln\lvert x+Y\rvert$	2	1/2	$\frac{1}{8}x(2x^2+a^2)Y-\frac{1}{8}a^4\ln\lvert x+Y\rvert$
0	3/2	$\frac{1}{8}x(2x^2+5a^2)Y+\frac{3}{8}a^4\ln\lvert x+Y\rvert$	3	-3/2	$Y+a^2/Y$
			3	-1/2	$\frac{1}{3}Y^3-a^2Y$
			3	1/2	$\frac{1}{5}Y^5-\frac{1}{3}a^2Y^3$

$I=\int\sin^m ax\cos^n ax\,dx$

m	*n*	*I*	*m*	*n*	*I*
1	0	$-(1/a)\cos ax$	1	1	$(\sin^2 ax)/2a$ of $-(\cos^2 ax)/2a$
0	1	$(1/a)\sin ax$	2	2	$-\frac{1}{32a}\sin 4ax+\frac{x}{8}$
1	-1	$-(1/a)\ln\lvert\cos ax\rvert$	1	n	$-\frac{\cos^{n+1}ax}{(n+1)a}$
-1	1	$(1/a)\ln\lvert\sin ax\rvert$	m	1	$\frac{\sin^{m+1}ax}{(m+1)a}$
2	0	$\frac{1}{2}x-\frac{1}{4a}\sin 2ax$	0	2	$\frac{1}{2}x+\frac{1}{4a}\sin 2ax$
3	0	$-\frac{1}{3a}\cos ax(\sin^2 ax+2)$	0	3	$\frac{1}{3a}\sin ax(\cos^2 ax+2)$
4	0	$\frac{3x}{8}-\frac{\sin 2ax}{4a}+\frac{\sin 4ax}{32a}$	0	4	$\frac{3x}{8}+\frac{\sin 2ax}{4a}+\frac{\sin 4ax}{32a}$

6.5 Benaderingen voor $\lvert x\rvert\to 0$

$(1+x)^a$	$1+ax+\ldots$	$\sin x$	$x-x^3/6+\ldots$
e^x	$1+x+\ldots$	$\cos x$	$1-x^2/2+\ldots$
$\ln(1+x)$	$x-x^3/2+\ldots$		