E&M Full content
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Boller, Offerhaus, Dhallé

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Fields and electromagnetism

Instructions

You have **3 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in.

You may use a hand-written formula sheet containing maximum 20 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (15pts/100) Below you find eight statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

1.a When we double the charge q of each of two identical point charges that are placed some distance apart, the electric field in-between them quadruples.

1.b In the vicinity of an infinitely long straight line that carries a homogeneous charge density λ, the electric field strength $E(s)$ decreases proportional to $1/s^2$, with s the perpendicular distance to the line;

1.c If at some location on an electric equipotential surface, **n** is the unit vector perpendicular to the surface and **E** is the electric field at the same location, then $\mathbf{E} \bullet \mathbf{n} = 0$.

1.d Consider an interface between a vacuum and a linear dielectric with relative permittivity $\varepsilon_r > 1$, without free surface charge. The electric field inside the dielectric then 'breaks away' from the normal to the interface (i.e. $\theta_{diel} \geq \theta_{vac}$, with θ the angle between the field and the normal).

1.e Two long and straight current-carrying wires that cross each other (some distance apart) at an angle of exactly 90° do not exert a Lorentz force on each other.

1.f In a static situation, the divergence of the curl of a magnetic vector potential **A** is always zero.

1.g A diamagnetic material that is placed near the end of a coil is attracted into the coil, while a paramagnetic material is pushed away from the coil.

1.h A closed copper ring is placed on top of a short solenoid that carries no current. When the current in the solenoid is switched on abruptly, the copper ring is repelled away from the solenoid.

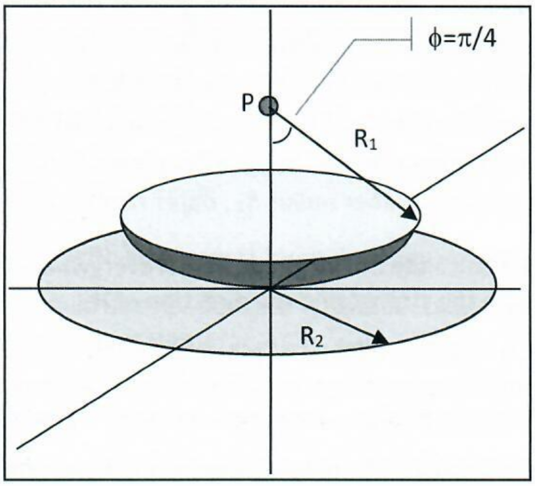
Problem 2 (20pts/100)

Consider a cap-shaped segment of a spherical surface that can be described as:

$A = \int_0^{\frac{\pi}{4}} \int_0^{2\pi} R_1^2 \sin(\varphi) d\vartheta d\varphi$, as in the figure.

This surface is homogeneously covered with a surface charge density σ.

2.a Calculate the **E**-field due to this cap at the center of the sphere (i.e. in the point P).



Now replace the cap by a flat circular disc with radius R_2 that intersects the vertical axis at the same point as the spherical cap. Suppose that this disk is covered with the same surface charge density σ .

2.b How big should the radius R_2 of this disc be to get the same electric field **E** in the point P?

If we choose the electric potential to be zero at infinity, $V(\infty) = 0$, the potential of a surface charge distribution $\sigma(\mathbf{r'})$ can be written as

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\boldsymbol{r}')}{\mathscr{r}} da' .$$

2.c Use this expression to calculate the potential $V_{cap}(P)$ due to the charged cap from question 2.a and $V_{disc}(P)$ due to the disc with radius R_2 from question 2.b (both calculated in the center point P).

2.d Which of the two potentials $V_{cap}(P)$ or $V_{disc}(P)$ is larger? Based on physical arguments, is this what you would expect?

Problem 3 (15pt/100)

An empty parallel-plate capacitor is charged until the potential difference between the plates reaches the value V_1 and then it is disconnected from the power supply. Next, a slab of dielectric material (with a relative permittivity ε_r) is inserted in-between the plates, in such a way that it fills the capacitor completely. The plates have a surface area A and are separated by a distance d.

3.a Calculate the new value V_2 of the potential difference between the plates;

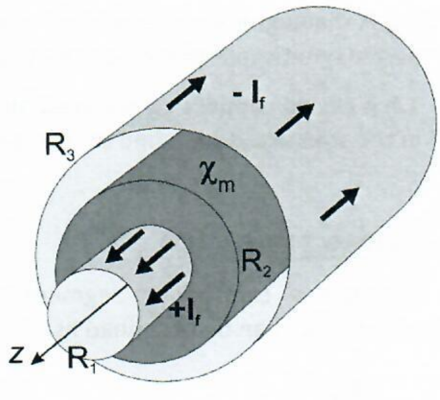
3.b Compare the electrostatic energy stored in the capacitor before and after the insertion of the dielectric;

3.c Based on your answer to question 3.b, reason whether the electrostatic force pulls the dielectric into a capacitor, or whether it tries to expel it.

Problem 4 (15pt/100)

Consider two infinitely long thin-walled conducting tubes with radii R_1 and R_3 that are both centred on the z - axis.

The inner conductor carries a total current I_f in the positive z - direction that is distributed uniformly over its circumference. That same total current returns uniformly distributed over the outer conductor. A thick shell of a material with a homogeneous magnetic susceptibility $\chi_m > 0$ is placed tightly around the inner conductor (inner radius R_1 , outer radius $R_2 < R_3$).

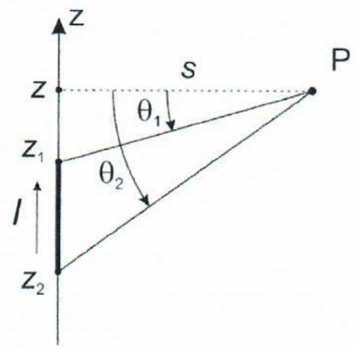


4.a Calculate the magnetic field **H** everywhere in space (i.e. $0 < s < R_1$; $R_1 < s < R_2$; $R_2 < s < R_3$ and $R_3 < s$). Give both the strengt *and* the direction of **H**.

4.b Calculate all the surface- and volumetric bound currents $\mathbf{K_B}$ en $\mathbf{J_B}$. Make a sketch in which you clearly indicate where they are located, how big they are and in which direction they flow. Also calculate the net total bound current flowing in the z-direction.

4.c Calculate the magnetic induction **B** everywhere (again both the strength *and* the direction).

Problem 5 (20pt/100)



5.a Consider a straight wire segment that carries a current *I*. Take the current direction as the *z*-direction of a cylindrical coordinate system and assume that the begin- and end-points of the segment to lie at $z = z_1$ and $z = z_2$, respectively.

Show that the magnetic induction **B** in an arbitrary point P(*s*,*z*) at a perpendicular distance *s* from z-axis can be written as:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)\hat{\boldsymbol{\phi}} ,$$

with θ_1 and θ_2 the angles defined in the figure and $\hat{\boldsymbol{\phi}}$ the usual azimuthal unit vector.

5.b Use the general expression

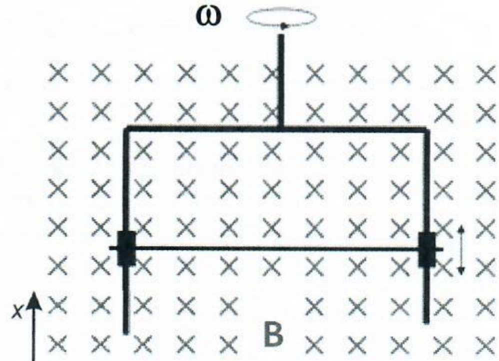
$$\mathbf{A} = \frac{\mu_0 I}{4\pi s}\int \frac{1}{\mathscr{r}}\,\mathrm{d}\boldsymbol{\ell}'$$

to work out the magnetic vector potential **A** in the arbitrary point P(*s*,*z*).

5.c With your answers to questions 5.a and 5.b, verify the relation

$$\mathbf{B} = \nabla \times \mathbf{A} \ .$$

Problem 6 (15pt/100)



The figure to the right shows a homogeneous magnetic field in the *z* - direction (pointing 'into the page').

Placed inside this field there is an electrically conducting wireframe that can rotate around one axis (*x*). One of the sides of the frame (the bottom side parallel to *y* in the picture) can slide in the $\pm x$ - direction along the adjoining sides.

The wireframe is rotated aroud the *x* - axis at a constant angular speed ω. Describe what happens over at least one full rotation: explain what happens to the magnetic flux throught the frame; to the induced current in the frame; and with any Lorentz force that may occur.

Explain if (and how) the free arm of the wireframe would move. Consider the direction and phase/timing of the movement (with respect to the phase of the rotation).