UNIVERSITEIT TWENTE.

I

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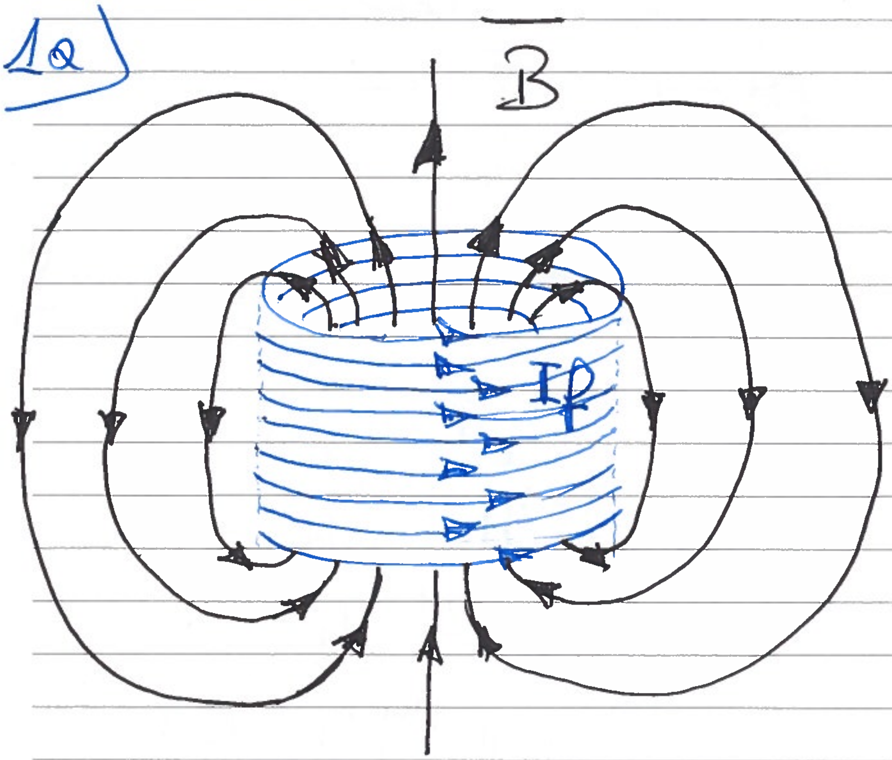
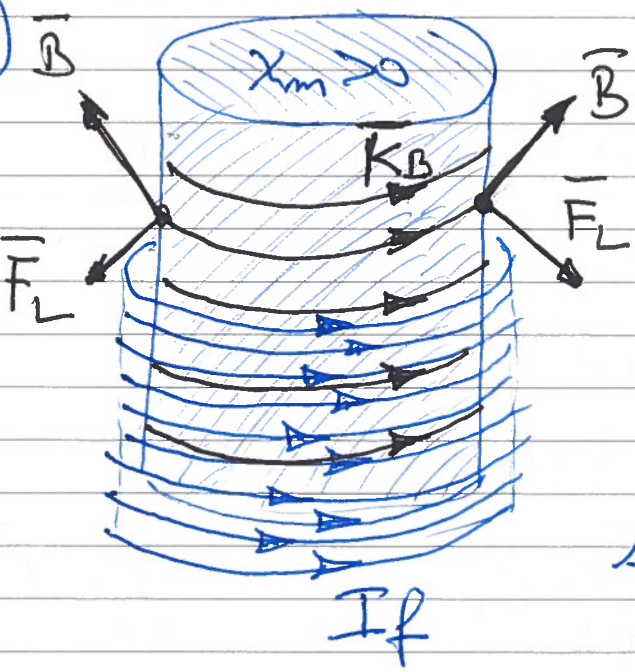
1a)



* $\overline{B}$-lines close
* Direction (inside +, outside −) + R.H.R.
* Not crossing
* Symmetry
* density inside ~ uniform.

1b)



* Position & direction $\overline{K_B}$

$\rightarrow \overline{M} = \chi_m \overline{H} = \chi_m H \hat{z}$

$\rightarrow \overline{K_B} = \overline{M} \times \hat{n} = \chi_m H (\hat{z} \times \hat{s})$

$= \chi_m H \hat{\phi}$

1c) $\overline{F_L} = \overline{K_B} \times \overline{B}$

points outwards & down

$\rightarrow$ $\boxed{\text{pulled in}}$

II

2a) NT, they attract each other
(2x right-hand rule)

2b) NT, $\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$

"Static" means no time-evolution

$\rightarrow \bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t} = 0$, but

this does <u>not</u> imply $\bar{J} = 0$

2c) NT, they bend away from the normal

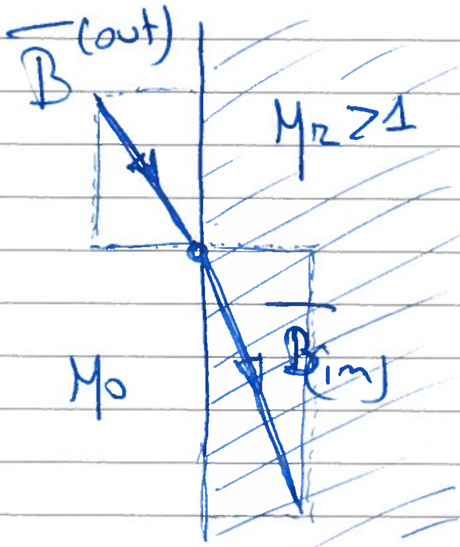
$$B_{\perp}^{(out)} = B_{\perp}^{(in)}$$

and $H_{//}^{(out)} - H_{//}^{(in)} = K_f = 0$

$$\rightarrow H_{//}^{(out)} = H_{//}^{(in)}$$

<u>But</u> $B_{//}^{(out)} = \mu_0 H_{//}^{(out)}$

$$< B_{//}^{(in)} = \mu_0 \mu_r H_{//}^{(in)}$$



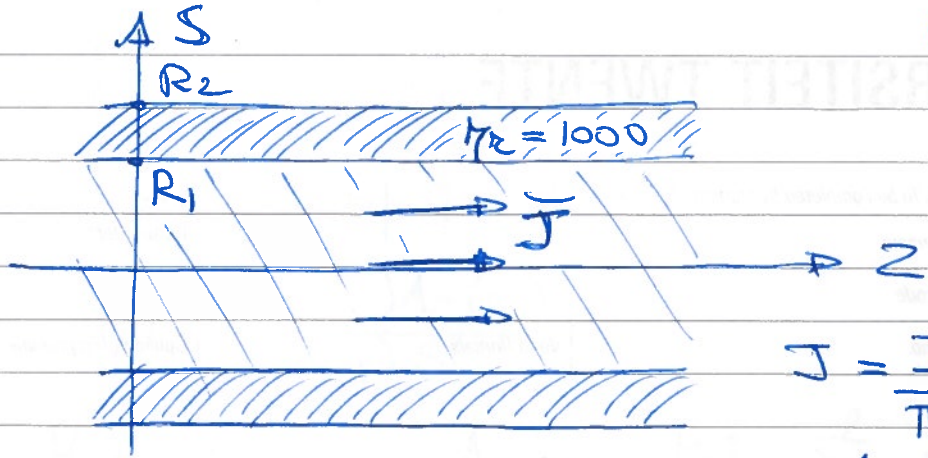
2d) NT, L is the ratio between the magnetic self-flux and the current ($\phi_M = LI$) and only depends on geometry and material properties.

2e) T, $U_M = \iiint \frac{B^2 d\tau}{2\mu_0} = \iiint \frac{\mu_r^2 \mu_0 H^2 d\tau}{2}$

and $H \propto I$ (Ampère)

III

5a)



$$J = \frac{I_0}{\pi R_1^2}$$

Symmetry $\rightarrow \bar{B} = B_\phi(s)\hat{\phi}$

$s < R_1$ $\quad \oint \bar{B}\cdot d\bar{\ell} = \mu_0 I_{encl}$

Ampère-loop circle at $s < R_1$

$$\Longrightarrow \begin{cases} \oint \bar{B}\cdot d\bar{\ell} = 2\pi s\, B_\phi(s) \\ I_{encl} = I_0\left(\frac{s}{R_1}\right)^2 \end{cases}$$

$$\rightarrow \boxed{\bar{B} = \frac{\mu_0 I_0}{2\pi}\frac{s}{R_1^2}\hat{\phi}} \quad (s < R_1)$$

$R_1 < s < R_2$ $\quad \oint \bar{H}\cdot d\bar{\ell} = I_{f,encl} = I_0$

$$\rightarrow \bar{H} = \frac{I_0}{2\pi s}\hat{\phi}$$

and $\bar{B} = \mu_r \mu_0 \bar{H}$

$$\rightarrow \boxed{\bar{B} = \mu_r \frac{\mu_0 I_0}{2\pi}\frac{1}{s}\hat{\phi}} \quad (R_1 < s < R_2)$$

$R_2 < s$ $\quad$ Idem for $\bar{H}$, but $\mu_r = 1$

$$\boxed{\bar{B} = \frac{\mu_0 I_0}{2\pi}\frac{1}{s}\hat{\phi}} \quad (R_2 < s)$$

IV

5b) $\bar{A} = A_z(s)\,\hat{z}$ (current direction)

and $\bar{B} = B_\phi \hat{\phi} = \bar{\nabla} \times \bar{A}$

$\rightarrow B_\phi = -\dfrac{\partial A_z(s)}{\partial s}$

① $B_\phi = \dfrac{\mu_0 I_0}{2\pi}\dfrac{s}{R_1^2} \rightarrow A_z = -\dfrac{\mu_0 I_0}{2\pi}\dfrac{s^2}{2R_1^2} + A_1$ $(s<R_1)$

② $B_\phi = \mu_r \dfrac{\mu_0 I}{2\pi}\dfrac{1}{s} \rightarrow A_z = -\mu_r\dfrac{\mu_0 I}{2\pi}\ln(s) + A_2$ $(R_1<s<R_2)$

③ $B_\phi = \dfrac{\mu_0 I}{2\pi}\dfrac{1}{s} \rightarrow A_z = -\dfrac{\mu_0 I}{2\pi}\ln(s) + A_3$ $(R_2<s)$

(*) Choose e.g. $A=0$ at $s=0$ $\rightarrow A_1 = 0$

(*) Continuity of $\bar{A}$ at $s=R_1 \rightarrow$

$$-\frac{\mu_0 I_0}{2\pi}\frac{1}{2} = -\mu_r\frac{\mu_0 I}{2\pi}\ln R_1 + A_2$$

$$\rightarrow A_2 = \frac{\mu_0 I}{2\pi}\left(\mu_r \ln R_1 - \frac{1}{2}\right)$$

(*) Continuity of $\bar{A}$ at $s=R_2 \rightarrow$

$$\frac{\mu_0 I_0}{2\pi}\left[\mu_r \ln\left(\frac{R_1}{R_2}\right) - \frac{1}{2}\right] = -\frac{\mu_0 I_0}{2\pi}\ln R_2 + A_3$$

$$\rightarrow A_3 = \frac{\mu_0 I_0}{2\pi}\left[\mu_r \ln\left(\frac{R_1}{R_2}\right) + \ln R_2 - \frac{1}{2}\right]$$

$$\Rightarrow \boxed{\begin{aligned} \bar{A}_1 &= -\frac{\mu_0 I_0}{2\pi}\frac{s^2}{2R_1^2}\hat{z} \\ \bar{A}_2 &= \frac{\mu_0 I_0}{2\pi}\left[\mu_r \ln\left(\frac{R_1}{s}\right) - \frac{1}{2}\right]\hat{z} \\ \bar{A}_3 &= \frac{\mu_0 I_0}{2\pi}\left[\mu_r \ln\left(\frac{R_1}{R_2}\right) + \ln\left(\frac{R_2}{s}\right) - \frac{1}{2}\right]\hat{z} \end{aligned}}$$

V

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5c) $\bar{H} = \frac{I_0}{2\pi s}\hat{\phi}$

$\bar{M} = (\mu_r - 1)\bar{H} = (\mu_r - 1)\frac{I_0}{2\pi s}\hat{\phi}$

$\bar{K}_B = \bar{M} \times \hat{n}$

2 surfaces ($s = R_1$ & $s = R_2$)

* $s = R_1 \rightarrow \hat{n} = -\hat{s}$

$\bar{K}_B = (\mu_r - 1)\frac{I_0}{2\pi R_1}(\hat{\phi} \times -\hat{s})$

$$\boxed{\bar{K}_B = (\mu_r - 1)\frac{I_0}{2\pi R_1}\hat{z}}$$

* $s = R_2 \rightarrow \hat{n} = +\hat{s}$

$$\boxed{\bar{K}_B = -(\mu_r - 1)\frac{I_0}{2\pi R_2}\hat{z}}$$

$\bar{J}_B = \bar{\nabla} \times \bar{M}$ with $\bar{M} = M_\phi(s)\hat{\phi}$

$\rightarrow \bar{J}_B = \frac{1}{s}\frac{\partial}{\partial s}\left[s M_\phi(s)\right] = \frac{1}{s}\frac{\partial}{\partial s}\left[\frac{(\mu_r - 1)I_0}{2\pi}\right]$

$$\rightarrow \boxed{\bar{J}_B = 0}$$